INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #23 — 10' 14 May 3, 2017

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Exercise 1. Let E and F be vector spaces over $\mathbb{K}$ and let $f : E \to F$ be a function. Recall the definition or a characterization of "f is a linear map."

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f is a linear map if it follows the characterisation:

$\forall u, v \in E$ and $\forall \lambda \in \mathbb{K}$, $f(u+\lambda v) = f(u) + \lambda f(v)$ ✓

Exercise 2. Determine the value of the following limit:

$$\ell = \lim_{x\to 0} \frac{1}{x^4}\left(e^{\sin(x)} - 1 - x - \frac{x^2}{2}\right).$$

If this limit doesn't exist, cross out the equal sign and write "DNE."

$\ell = -\dfrac{1}{24}$ 0

Exercise 3. Let $E = \mathbb{R}^3$ and let $\mathscr{B} = (u_1, u_2, u_3)$ where

$u_1 = (1, 0, 0)$, $u_2 = (1, 1, 0)$, $u_3 = (1, 1, 1)$.

You're given that $\mathscr{B}$ is a basis of E.

1. Let $\alpha \in \mathbb{R}$. Compute $u_1 + u_2 + \alpha u_3$.

3 $u_1 + u_2 + \alpha u_3 = (\alpha + 2, \alpha + 1, \alpha)$ ✓

2. Deduce the coordinates of $(5, 4, 3)$ in the basis $\mathscr{B}$.

5 $[(5,4,3)]_{\mathscr{B}} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$ ✓

$\sin(x) \underset{x\to 0}{=} x - \dfrac{x^3}{6} + o(x^4)$ $= 1 + x + \dfrac{x^2}{2} - \dfrac{x^4}{24}$

$e^{\sin(x)} - 1$

$e^x \underset{x\to 0}{=} 1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{6} + \dfrac{x^4}{24} + o(x^4)$

$= 1 + x - \dfrac{x^3}{6} + \dfrac{x^2}{2} - \dfrac{x^4}{12} + \dfrac{x^3}{6} + \dfrac{x^4}{24} + o(x^4)$