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INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #11 — 10'

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Exercise 1. Let f and g be two functions defined in a neighborhood V of $x_0 \in \mathbb{R}$ such that:

- $f(x_0) = 0$,
- f is differentiable at x_0,
- g is continuous at x_0.

Show that the product function fg is differentiable at x_0 and determine the value of $(fg)'(x_0)$.

$x_0 \in \mathbb{R}$, $h \in \mathbb{R}^*$

$$\frac{f(x_0+h)\,g(x_0+h) - f(x_0)\,g(x_0)}{h}$$ as f and g continuous at x_0 ⇐ (differentiable at x_0 so continuous)

1 $$= f(x_0+h)\,\frac{g(x_0+h)-g(x_0)}{h} + g(x_0)\,\frac{f(x_0+h)-f(x_0)}{h}$$

$$\xrightarrow[h\to 0]{} f(x_0)\,g'(x_0) + g(x_0)\,f'(x_0) = (fg)'(x_0)$$

$f(x_0)=0$ hence $(fg)'(x_0) = g(x_0)\,f'(x_0)$

g is NOT necessarily differentiable

Exercise 2. Compute the derivative of the function

$$f : \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto \frac{\sin(x)\cos(x)}{2+\cosh(x)\sqrt{|x|}} + 1$$

at 0. No justifications required.

4 $f'(0) = \frac{1}{2}$

Exercise 3. Compute the derivative of the function

$$f : \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto \sin\left(e^{x\cos(x)}\right).$$

No justifications required.

5 $\forall x \in \mathbb{R},\ f'(x) = (\cos(x) - x\sin(x))\,e^{x\cos(x)}\cos\left(e^{x\cos(x)}\right)$

Exercise 4. Determine limits (in $\overline{\mathbb{R}}$) of sequences $(u_n)_n$ defined below; if a limit does not exist, cross out the equal sign and write "DNE."

1	$\forall n \in \mathbb{N},\ u_n = \frac{1}{2n+3}$. $= 0$	$\lim_{n\to+\infty} u_n = 0$
1	$\forall n \in \mathbb{N},\ u_n = \frac{n+2}{n+3}$.	$\lim_{n\to+\infty} u_n = 1$
2	$\forall n \in \mathbb{N}^*,\ u_n = \left(1+\frac{2}{n}\right)^n$.	$\lim_{n\to+\infty} u_n = e^2$
2	$\forall n \in \mathbb{N},\ u_n = n^2 + \sin(n)n$.	$\lim_{n\to+\infty} u_n = +\infty$