17

INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #14 — 10' February 14, 2018

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Exercise 1. Fill in the blanks with the Taylor–Young expansion at the appropriate order (without the $\sum$ symbol):

4

$$\cos(x) \underset{x\to 0}{=} 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + o(x^7)$$ ✓

$$\frac{1}{1-x} \underset{x\to 0}{=} 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + o(x^6)$$ ✓

and, for $n \in \mathbb{N}^*$, give the general formula (with the $\sum$ symbol) of the following Taylor–Young expansion:

2

$$e^x \underset{x\to 0}{=} \sum_{k=0}^{n} \frac{x^k}{k!} + o(x^n)$$ ✓

Exercise 2. Determine the simplest equivalents (no justifications required):

3

$$\sinh(x)\left(\cos\left(\frac{x}{2}\right) - 1\right) \underset{x\to 0}{\sim} -\frac{x^3}{8}$$ ✓

$$\frac{e^x - 1 - x}{\arctan(x)} \underset{x\to 0}{\sim} \cancel{1} \qquad \frac{\frac{x^2}{2!}}{x} = \frac{x}{2}$$

Exercise 3. Fill in the blank with the Taylor–Young expansion at the specified order.

4

$$\ln(1+x)\sin(x) \underset{x\to 0}{=} x^2 - \frac{x^3}{2} + \frac{x^4}{3} - \frac{x^4}{3!} + o(x^4) = x^2 - \frac{x^3}{2} + \frac{x^4}{6} + o(x^4)$$ ✓

$$\ln(1+\sin(x)) \underset{x\to 0}{=} x - \frac{x^3}{3!} - \frac{1}{2}\left(x - \frac{x^3}{3!}\right)^2 + \frac{1}{3}\left(x - \frac{x^3}{3!}\right)^3 - \frac{1}{4}\left(x - \frac{x^3}{3!}\right)^4 + o(x^4)$$

$$= x - \frac{x^3}{3!} - \frac{x^2}{2} + \frac{x^4}{3!} + \frac{x^3}{3} - \frac{1}{4}x^4 + o(x^4)$$

4

$$= x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + o(x^4)$$ ✓