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INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #15 — 12'

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Exercise 1. Let f be a function 3 times differentiable at 0 such that

$$f(x) \underset{x\to 0}{=} 1 - x + 5x^2 - x^3 + o(x^3).$$

$\frac{f^{(k)}(x_0)(x)^k}{k!} = -x^3$

$f'(2) =$

Determine the value of $f^{(k)}(0)$, for $k \in \{0,1,2,3\}$:

6

$f(0) = 1$	$f'(0) = -1$
$f''(0) = 5 \times 2$	$f'''(0) = -1 \times 6$

Exercise 2. For which value of $\alpha \in \mathbb{R}$ is the following equality true?

$$\int_0^1 \frac{1}{1+x^3}\,dx = \lim_{n\to+\infty} n^\alpha \sum_{k=1}^{n} \frac{1}{n^3+k^3}.$$

No justifications required.

0

$\alpha =$

Exercise 3. Fill in the blanks with the Taylor–Young expansions at the specified order:

$\frac{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}}{x}$

2

$$\frac{\ln(1+X)}{X} \underset{X\to 0}{=} 1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \frac{x^4}{5} + o(X^4)$$

0

$$\frac{\ln(1+\sinh(x))}{\sinh(x)} \underset{x\to 0}{=} 1 - \frac{\sinh(x)}{2} + \frac{\sinh(x)^2}{3} - \frac{(\sinh)^3}{4} + \frac{\sinh(x)^4}{5} + o(x^4)$$

$=$

Exercise 4. Let $f : [0,1] \to \mathbb{R}$ be a continuous function and let $T = ((x_0,x_1,x_2,x_3),(t_1,t_2,t_3))$ be the tagged subdivision of $[0,1]$ where

$x_0 = 0$ $\quad x_1 = 0.3$ $\quad x_2 = 0.8$ $\quad x_3 = 1$

$t_1 = 0.2$ $\quad t_2 = 0.65$ $\quad t_3 = 0.9$

Write the Riemann sum $R(f,T)$ of f associated to the tagged subdivision T:

0

$= t_1(x_1 - x_0) + t_2(x_2 - x_1) + t_3(1 - 0{,}8)$

$R(f,T) =$ $0{,}2(0{,}3-0) + 0{,}65(0{,}8-0{,}3) + 0{,}9 \times (1-0{,}8)$

$= f(0{,}2) \times 0{,}3 + f(0{,}65) \times 0{,}5 + f(0{,}9) \times 0{,}2$

$= \sum_{k=1}^{3} f(t_k) \times (x_k - x_{k-1})$