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INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #16 — 12'

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Exercise 1. Fill in the blank with the Taylor–Young expansion at the specified order:

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$$\frac{e^{\sinh(x)}-1}{\sinh(x)} \underset{x\to 0}{=} \cancel{1} + \frac{x}{2} - \frac{x^3}{\cancel{8}\,12} + \left(x-\frac{x^3}{6}\right)^2 \times \frac{1}{6} + \left(x-\frac{x^3}{6}\right)^3 \times \frac{1}{24} + \left(x-\frac{x^3}{6}\right)^4 \times \frac{1}{5!} + o(x^4)$$

yes! sorry

Exercise 2. The following limit is the limit of a Riemann sum of a certain continuous function associated with a certain tagged subdivision of a certain interval. Recognize the value of the limit as an integral (fill in the blanks). No justifications required.

1

$$\lim_{n\to+\infty} n^2 \sum_{k=1}^{n} \frac{2k}{2n^4+k^4} = \int_0^1 \frac{2x}{2+x^4}\,dx$$

Exercise 3. Recall the Mean Value Theorem for integrals (MVT2).

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Let : f a continuous function on $[a,b]$

g a positive, piecewise continuous function on $[a,b]$

Then there exists $c \in [a,b]$ s.t.

$$\int_a^b f(x)\,g(x)\,dx = f(c)\int_a^b g(x)\,dx$$

Exercise 4. Use an integration by parts to compute the value of the following integral:

$$I = \int_0^1 x\sinh(x)\,dx$$

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$$I = [\cancel{-}x\cosh(x)]_0^1 + \int_0^1 1\times\cosh(x) = -\cosh(1) + [\sinh(x)]_0^1$$
$$= \sinh(1) - \cosh(1) = ?$$