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INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #17 — 10' March 21, 2018

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Exercise 1. Fill in the blank with the Taylor–Young expansion at the specified order:

$$\ln(1+2\ln(1+x)) \underset{x\to 0}{=} \ln\left(1+2\left(x-\frac{x^2}{2}+\frac{x^3}{3}\right)\right) + o(x^3)$$

$$= \ln\left(1+2x-x^2+\frac{2x^3}{3}\right)$$

$$= 2x-x^2+\frac{2x^3}{3} - \frac{1}{2}\left(2x-x^2+\frac{2x^3}{3}\right)^2 + \frac{1}{3}\left(2x-x^2+\frac{x^3}{3}\right)^3 + o(x^3)$$

$$= 2x-x^2+\frac{2x^3}{3} - 2x^2+2x^3+\frac{8}{3}x^3+o(x^3) = 2x-3x^2+\frac{16x^3}{3}+o(x^3)$$

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Exercise 2. Solve the following linear system using the Gaussian elimination. You will explicitly mention the elementary row operations you're performing at each step of the descent.

$$(S)\begin{cases} -x+y+z=-1 \\ 2x-y-2z=2 \\ -3x+y+4z=1 \end{cases} \iff \begin{matrix} R_2\leftarrow R_2+2R_1 \\ R_3\leftarrow R_3-3R_1 \end{matrix} \begin{cases} -x+y+z=-1 \\ y=0 \\ -2y+z=4 \end{cases}$$

$$\iff \begin{matrix} \\ R_3\leftarrow R_3+2R_2 \end{matrix} \begin{cases} -x+y+z=-1 \\ y=0 \\ z=4 \end{cases} \iff \begin{cases} x=5 \\ y=0 \\ z=4 \end{cases}$$

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What is the rank of the system (S)?

$\mathrm{rk}(S) =$ 3

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Exercise 3. Use the substitution

$$x = \sin t$$

to compute the value of the following integral:

$$I = \int_0^{\sqrt{3}/2} \sqrt{1-x^2}\,\mathrm{d}x$$

$$I = \int_0^{\frac{\pi}{3}} \sqrt{1-(\sin(t))^2}\cos(t)\,dt = \int_0^{\frac{\pi}{3}} \cos^2(t)\,dt = \left[\frac{1}{3}\sin^3(t)\right]_0^{\frac{\pi}{3}}$$

$$= \frac{1}{3}*\left(\frac{\sqrt{3}}{2}\right)^3$$

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