INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #18 — 12' 17 March 28, 2018

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Exercise 1. Determine the Taylor–Young expansion of the following expression at the specified order:

$$\ln(\cos(x)+x) \underset{x\to 0}{=} \ln\left(1-\frac{x^2}{2}+\frac{x^4}{4!}+x\right) + o(x^4)$$

$$= x - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{1}{2}\left(x-\frac{x^2}{2}\right)^2 + \frac{1}{3}\left(x-\frac{x^2}{2}\right)^3 - \frac{1}{4}x^4 + o(x^4)$$

$$= x - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^2}{2} + \frac{x^3}{2} - \frac{x^4}{8} + \frac{x^3}{3} - \frac{x^4}{2} - \frac{x^4}{4} + o(x^4)$$

$$= x - x^2 + \frac{5x^3}{6} - \frac{5}{6}x^4 + o(x^4)$$

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Exercise 2. Let $E = \mathbb{R}^{\mathbb{R}}$ be the vector space of all functions from $\mathbb{R}$ to $\mathbb{R}$ and let

$$F = \{f \in E \mid f(0) + f(1) = 0\}.$$

Show that F is a subspace of E.

prove it!

• $0_E \in F$ Hence $F \neq \emptyset$

Let $f, g \in F$ $\lambda, \mu \in \mathbb{R}$

$(\lambda f + \mu g)(0) + (\lambda f + \mu g)(1) = \lambda(f(0) + f(1)) + \mu(g(0) + g(1))$

$= 0 + 0 = 0$ $\in F$ who?

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Exercise 3. Let E be a vector space over $\mathbb{K}$ and let F be a subset of E. Recall the definition of

"F is a subspace of E."

• $F \neq \emptyset$

? $\mathbb{K}$ or $\mathbb{R}$?

• $\forall u, v \in F$, $\forall \lambda, \mu \in \mathbb{K}$, $\lambda u + \mu v \in F$

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Exercise 4. Let

$$F = \{(x,y,z) \in \mathbb{R}^3 \mid x + y = 1\}.$$

Is F a subspace of $E = \mathbb{R}^3$? justify as concisely as possible.

No : $(0,0,0) \notin F$ OK but prove it!

Let (x,y,z) and $(x',y',z') \in F$, $\lambda, \mu \in \mathbb{R}$

$\lambda x + \mu x' + \lambda y + \mu y' = \lambda(x+y) + \mu(x'+y')$

$= \lambda + \mu \neq 1$

useless!

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