20

INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #20 — 12'

May 2, 2018

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Exercise 1. Let E and F be vector spaces over $\mathbb{K}$ and let $f : E \to F$ be a function. Recall the definition or a characterization of "f is a linear map."

$\forall u, v \in E, \forall \lambda \in \mathbb{K}, f(u + \lambda v) = f(u) + \lambda f(v)$ ✓

Exercise 2. Determine the Taylor–Young expansion at 0 of the function

$$f : \begin{array}{ccc} (-\pi/4, \pi/4) & \longrightarrow & \mathbb{R} \\ x & \longmapsto & \ln(\cos(x) + x) \end{array}$$

at 0 at the specified order:

8

$f(x) \underset{x \to 0}{=} \ln\left(1 - \frac{x^2}{2} + \frac{x^4}{4!} + x\right) + o(x^4)$ $\qquad +o(x^4)$

$= x - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{\left(x - \frac{x^2}{2} + \frac{x^4}{4!}\right)^2}{2} + \frac{\left(x - \frac{x^2}{2} + \frac{x^4}{4!}\right)^3}{3} - \frac{x^4}{4} + o(x^4)$

$= x - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^4}{4} - \frac{x^2}{2} - \frac{x^4}{8} + \frac{x^3}{2} + \frac{x^3}{3} - \frac{x^4}{2} + o(x^4) = x - x^2 + \frac{5x^3}{6} - \frac{5}{6}x^4 + o(x^4)$ ✓

Exercise 3. Let $f : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be a linear map such that

$$f(1,0,1) = (1,2), \qquad f(1,2,1) = (3,1).$$

Determine the value of $f(-1,2,-1)$.

$f(-1,2,-1) = (1, -3)$ ✓

Exercise 4. Let E and F be vector spaces over $\mathbb{K}$ and let $f : E \to F$ be a linear map. Recall the definition of the kernel and the image of f:

4

$\operatorname{Ker} f = \{u \in E \mid f(u) = 0_F\}$ ✓

$\operatorname{Im} f = f(E)$ ✓