$-1\ -2\ +3$
$2\ +1\ -2$
$-2\ -2\ +3\ +1$

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INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #23 — 12'

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$5\ \ 1\ +1\ 0\ +3$
$-1\ -2$
$2\ +1\ +1$

Name: Noëlie Giron

Exercise 1. Determine the value of the following limit:

$$\ell = \lim_{x\to 0} \frac{\sinh(e^x - 1) - x - x^2/2 - x^3/3}{x^4}.$$

If this limit doesn't exist, cross out the equal sign and write "DNE."

$$\ell = \boxed{\frac{7}{24}} = \frac{1}{24} + \frac{6}{24} = \frac{1}{24} + \frac{1}{4} = \frac{1}{24} + \frac{3}{6} \times \frac{1}{2}$$

$-1\ -2\ +3$ $\quad -2\ -2\ +3\ +1$ $\quad 2\ +1\ +1$
$1\ +1\ +3$

Exercise 2. Compute the product of matrices below. If the product doesn't exist, cross out the equal sign and write DNE.

$$\begin{pmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & 1 & -2 \\ 2 & -1 & 3 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 2 & -1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ 1 & -3 \\ 0 & 4 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 1 \\ 2 & -1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & 1 & -2 \\ 2 & -1 & 3 & 1 \end{pmatrix} \cancel{=} \text{DNE}$$

Exercise 3. Let E be a vector space of dimension 2 and let $\mathscr{B} = (e_1, e_2)$ be a basis of E. Let $f : E \to E$ be the endomorphism of E such that

$$f(e_1) = 2e_1 + e_2, \qquad f(e_2) = e_1 + 2e_2.$$

You don't need to justify that such an endomorphism exists and is unique.

1. Compute $f^2(e_1)$ and $f^2(e_2)$.

$f^2(e_1) = f\circ f(e_1) = f(2e_1+e_2) = 2f(e_1) + f(e_2) = 2\times(2e_1+e_2) + e_1+2e_2 = 5e_1+4e_2$

$f^2(e_2) = f\circ f(e_2) = f(e_1+2e_2) = f(e_1) + 2f(e_2) = 2e_1+e_2 + 2\times(e_1+2e_2) = 4e_1+5e_2$

2. Let $P \in \mathbb{R}[X]$ be the polynomial $P = X^2 - 4X + 3$. Compute $P(f)(e_1)$ and $P(f)(e_2)$.

$3e_1$

$P(f)(e_1) = (2e_1+e_2)^2 - 4(2e_1+e_2) + 3\,\mathrm{id}(f) = 4e_1^2 + 4e_1e_2 + e_2^2 - 8e_1 - 4e_2 + 3\,\mathrm{id}(f)$

$P(f)(e_2) = (e_1+2e_2)^2 - 4(e_1+2e_2) + 3\,\mathrm{id}(f) = e_1^2 + 4e_1e_2 + 4e_2^2 - 4e_1 - 8e_2 + 3\,\mathrm{id}(f)$

$3e_2$