INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

20

SCAN 1 — Quiz #2 — 10' *September 27, 2017*

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Exercise 1. Let $x \in \mathbb{R}$. Recall the half-angle formula:

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$ ✓

Exercise 2. Let $x, y \in \mathbb{C}$ and $n \in \mathbb{N}$. Recall the Binomial Theorem:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$ ✓

Exercise 3. Let A be a non-empty subset of $\mathbb{R}$ and let $f : A \to \mathbb{R}$ be a function. Recall the definition of "f is decreasing."

$$\forall x, y \in A, \ (x < y \Rightarrow f(x) > f(y))$$ ✓

Exercise 4. Let $x \in \mathbb{C}$. Expand:

$$(x-2)^5 = x^5 - 5x^4 \times 2 + 10x^3 \times 4 - 10x^2 \times 8 + 5x \times 16 - 32$$
$$= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$$ ✓

Exercise 5. Let $x \in \mathbb{R}$. Fill in the blank (only give your answer, no justifications required):

$$\cos(3x) = \frac{1}{2} \iff \cos(3x) = \cos\left(\frac{\pi}{3}\right) \iff \exists k \in \mathbb{Z}, \ 3x = \frac{\pi}{3} + 2k\pi \text{ or } 3x = -\frac{\pi}{3} + 2k\pi$$
$$\iff \exists k \in \mathbb{Z}, \ x = \frac{\pi}{9} + \frac{2}{3}k\pi \text{ or } x = -\frac{\pi}{9} + \frac{2}{3}k\pi$$ ✓

Exercise 6. Fill in the blank:

$$\cos\left(\frac{29\pi}{3}\right) = \cos\left(\frac{30\pi}{3} - \frac{\pi}{3}\right) = \cos\left(10\pi - \frac{\pi}{3}\right) = \cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$$ ✓

Exercise 7. Let $n \in \mathbb{N}$. Simplify:

$$\frac{((n+1)!)^2}{(2n)!}\binom{2n+1}{n+1} = \frac{((n+1)!)^2}{(2n)!} \times \frac{(2n+1)!}{(n+1)!\,(n)!} = \frac{(n+1)! \times (2n+1)!}{(2n)! \times (n)!} = (n+1)(2n+1)$$ ✓