INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #4 — 12' 14

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Exercise 1. Give an example of a function $f : [0,1] \to \mathbb{R}$ that is injective and **not** monotone. Give the explicit formula for f and plot its graph.

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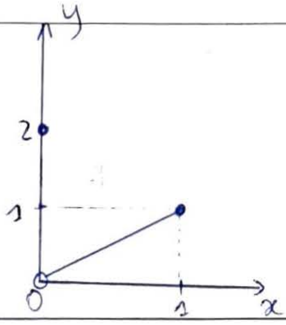
$$f : [0,1] \longrightarrow \mathbb{R}$$
$$x \longmapsto \begin{cases} x \text{ if } 0 < x \leq 1 \\ 2 \text{ if } x = 0 \end{cases}$$



Exercise 2. Let A be a non-empty subset of $\mathbb{R}$ symmetric with respect to 0, and let $f : A \to \mathbb{R}$. Recall the definition of "f is odd."

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$\forall x \in A, \ f(-x) = -f(x)$

Exercise 3. Linearize the following trigonometric expression:

$$\forall x \in \mathbb{R},\ \sin^4(x) = \left[\sin^2(x)\right]^2 = \left[\frac{\sin(2x)-1}{2}\right]^2 = \frac{\sin^2(2x) - 2\sin(2x) + 1}{4} = \frac{\frac{\sin(4x)-1}{2} - 2\sin(2x)+1}{2}$$
$$= \frac{\sin(4x) - 4\sin(2x) + 1}{4}$$

Exercise 4. Let

$$f : \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto 3x + 2.$$

Show that f is a bijection and determine f^{-1}. You may use the back of this sheet if your answer doesn't fit in the box below.

Let $x \in \mathbb{R}$, $y \in \mathbb{R}$.

$$f(x) = y \iff 3x + 2 = y$$
$$\iff 3x = y - 2$$
$$\iff x = \frac{y-2}{3}$$

5 Since $f(x) = y \Rightarrow x = \frac{y-2}{3}$ then f is injective. Clearly, $\frac{y-2}{3} \in \mathbb{R}$ so f is surjective. Therefore f is bijective and $f^{-1} : \mathbb{R} \to \mathbb{R}$
$$y \mapsto \frac{y-2}{3}$$

Exercise 5. Let $x, y \in \mathbb{R}$. Recall the product formula

$$\cos(x)\cos(y) = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right].$$

a? b?

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