INSA | INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #5 — 10'

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Name: Lionore Giner

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Exercise 1. Let $a \in \mathbb{R}$ and let f be the polynomial function defined by:

$$\begin{aligned} f : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto a(a-2)x^2 + ax - a + 7. \end{aligned}$$

Determine the degree of f. No justifications required.

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$$\deg(f) = \begin{cases} 0 \text{ if } a = 0 \\ 1 \text{ if } a = 2 \\ 2 \text{ if } a \in \mathbb{R} \setminus \{0;2\} \end{cases}$$

Exercise 2. Let f be the polynomial function defined as

$$\begin{aligned} f : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto x^4 + 2x^3 - 2x - 1. \end{aligned}$$

1. Show that 1 and -1 are roots of f.

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$f(1) = 1^4 + 2 \times 1^3 - 2 \times 1 - 1 = 1 + 2 - 2 - 1 = 0 \Leftarrow$ so 1 is a root of f

$f(-1) = (-1)^4 + 2 \times (-1)^3 - 2 \times (-1) - 1 = 1 - 2 + 2 - 1 = 0 \Leftarrow$ so -1 is a root of f

2. Determine the polynomial functions g such that

$$\forall x \in \mathbb{R}, \ f(x) = (x-1)(x+1)g(x).$$

No justifications required.

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$\forall x \in \mathbb{R}, \ g(x) = x^2 + 2x + 1 = (x+1)^2$

3. Deduce the roots of f and their multiplicities. No justifications required.

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The roots of f are -1 and 1

-1 is of multiplicity 3 and 1 of multiplicity 1

Exercise 3. Let $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. Let $f : \mathbb{K} \to \mathbb{K}$ be a polynomial function, let $x_0 \in \mathbb{K}$, and let $m \in \mathbb{N}^*$. Recall the definition of

"x_0 is a root of f of multiplicity m."

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$\forall x \in \mathbb{K}, \ f(x) = (x - x_0)^m g(x)$

and g is a polynomial function on $\mathbb{K}$ with $g(x_0) \neq 0$.