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Name: MAGNIER ZOE

Exercise 1.

1. Let $x_0 \in \mathbb{R}$ and let f be a function defined on a punctured neighborhood of x_0 . Recall the definition of

$$\lim_{x \rightarrow x_0} f(x) = -2.$$

$$\forall \varepsilon > 0, \exists \delta > 0, \forall x \in (x_0 - \delta, x_0) \cup (x_0, x_0 + \delta), |f(x) + 2| < \varepsilon \quad \checkmark$$

2. Let f be a function defined in a neighborhood of $-\infty$. Recall the definition of

$$\lim_{x \rightarrow -\infty} f(x) = +\infty.$$

$$\forall \varepsilon > 0, \exists n \in \mathbb{R}, \forall x \in (-\infty, n), f(x) > \varepsilon. \quad \checkmark$$

Exercise 2.

1. Give the general solution of the following differential equation:

$$-3f' + 2f = 0.$$

No justifications required.

The general solution is $f: \mathbb{R} \rightarrow \mathbb{R}$
 $x \mapsto A e^{\frac{2}{3}x}, A \in \mathbb{R}. \quad \checkmark$

2. Give the solution of the following initial value problem:

$$\begin{cases} -3f' + 2f = 0 \\ f(2) = 3. \end{cases}$$

No justifications required.

$$f(2) = 3 \Leftrightarrow A e^{\frac{2}{3} \cdot 2} = 3 \Leftrightarrow A \times e^{\frac{4}{3}} = 3 \Leftrightarrow A = \frac{3}{e^{\frac{4}{3}}} = 3e^{-\frac{4}{3}}.$$

Hence $\forall x \in \mathbb{R}, f(x) = 3e^{-\frac{4}{3}} e^{\frac{2}{3}x} = 3e^{\frac{2}{3}x - \frac{4}{3}} \quad \checkmark$