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SCAN 1 — Quiz #10 — 10'

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Name: CLAUS Grégoire

18/20

Exercise 1. Determine the value of the following limits. You're asked to write your answer in the following form:

$\forall$variable $\in$ appropriate neighborhood,

expression given = expression that enables you to compute the limit $\underset{variable \to point}{\longrightarrow}$ value of limit (or DNE).

a) $\lim\limits_{x\to 0} \dfrac{\ln(1+\sin(x))}{\ln(1-x)}$. $\underline{\dfrac{\ln(1+\sin(x))}{\sin(x)}}$

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$\forall x \in]-1;0[\cup]0;1[$

$$\frac{\ln(1+\sin(x))}{\ln(1-x)} = \frac{\ln(1+\sin(x))}{\sin(x)} \cdot \frac{-x}{\ln(1-x)} \cdot \frac{\sin(x)}{-x} \underset{x\to 0}{\longrightarrow} -1$$

b) $\lim\limits_{x\to 0} \dfrac{e^{\sin^2(x)}-1}{\sin(x^2)}$.

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$\forall x \in]-\sqrt{\pi};0[\cup]0;\sqrt{\pi}[$

$$\frac{e^{\sin^2(x)}-1}{\sin(x^2)} = \frac{e^{\sin^2(x)}-1}{\sin^2(x)} \times \left(\frac{\sin(x)}{x}\right)^2 \times \frac{x^2}{\sin(x^2)} \underset{x\to 0}{\longrightarrow} 1$$

c) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x^2}-1}{\cos(x)\sin(x^2)}$. $\sin(x^2) =$

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$\forall x \in]\sqrt{\frac{\pi}{2}};0[\cup]0;\sqrt{\frac{\pi}{2}}[$

$$\frac{\sqrt{1+x^2}-1}{\cos(x)\sin(x^2)} = \underbrace{\frac{x^2}{\sin(x^2)}}_{\underset{x\to 0}{\longrightarrow} 1} \times \underbrace{\frac{\sqrt{1+x^2}-1}{x^2}}_{\underset{x\to 0}{\longrightarrow} \cancel{\infty}} \times \underbrace{\frac{1}{\cos(x)}}_{\underset{x\to 0}{\longrightarrow} 1} \underset{x\to 0}{\longrightarrow} \cancel{\infty}$$

Exercise 2. Let

$$f : \mathbb{R} \longrightarrow \mathbb{R}, \quad x \longmapsto \cos(x)+2.$$

Show that there exists $x_0 \in \mathbb{R}$ such that $f(x_0) = x_0^2$.

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Let $g: \mathbb{R} \to \mathbb{R}$,
$x \mapsto \cos(x)+2-x^2$

g is continuous on $\mathbb{R}$ by sum of continuous functions on $\mathbb{R}$ $\Rightarrow g$ continous on $[0;\pi]$

Now: $g(0) = 1+2-1 = 2 > 0$

$g(\pi) = -1+2-\pi^2 = 1-\pi^2 < 0$, thus from Bolzano's theorem

$\exists\, x_0 \in [0;\pi]$, st $g(x_0)=0$, thus

$\exists\, x_0 \in [0;\pi]$, st $f(x_0) = x_0^2$