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INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #17 — 12' April 3, 2019

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Exercise 1. Determine an antiderivative F of the rational function f defined by:

$$\forall X \in \mathbb{R},\ f(X) = \frac{1}{X^2+X+2} = \frac{1}{(x+\frac{1}{2})^2+\frac{7}{4}} = \frac{\frac{4}{3}}{\frac{4}{3}(x+\frac{1}{2})^2+1}$$

$$= \frac{4}{3}\frac{1}{\left(\frac{2(x+\frac{1}{2})}{\sqrt{3}}\right)^2+1} = \frac{4}{3}\frac{1}{\frac{2x+1}{\sqrt{3}}}$$

$\forall x \in \mathbb{R},\ F(x) =$

$$F(x) = \frac{4}{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\times\frac{\sqrt{3}}{2} = \frac{2\sqrt{3}}{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)$$

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Exercise 2. Use the substitution $x = \cos(t)$ to compute the value of the following integral. You'll get partial credit if you only perform the substitution but can't compute the value of the integral.

$$I = \int_0^1 \sqrt{1-x^2}\,\mathrm{d}x = \int_{\frac{\pi}{2}}^{0}\sqrt{1-\arccos^2 x}\ -\sin(t)\,dt =$$

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Exercise 3. Give the partial fraction decomposition of the following rational fraction (no justifications required).

$$\frac{2X}{(X-2)(X^2+1)} = \frac{A}{(X-2)} + \frac{BX+C}{(X^2+1)} = \frac{\frac{4}{5}}{(X-2)} + \frac{\frac{9}{10}X + \frac{2}{5}}{(X^2+X)}$$

$A = \frac{4}{5}$ $C = \frac{2}{5}$ $B = \frac{9}{10}$

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Exercise 4. Use an integration by parts to compute the value of the following integral:

$$I = \int_1^e x\ln(x)\,\mathrm{d}x = \left[\ln(x)\frac{x^2}{2}\right]_1^e - \int_1^e \frac{x^2}{2}\cdot\frac{1}{x}$$

$u(x) = \ln(x)$
$u'(x) = \frac{1}{x}$
$v'(x) = x$
$v(x) = \frac{x^2}{2}$

$$= \ln(e)\frac{e^2}{2} - \ln(1)\frac{1}{2} - \int_1^e \frac{x}{2}\,dx$$

$$= \frac{e^2}{2} - \left[\frac{x^2}{4}\right]_1^e = \frac{e^2}{2} - e^2 + 1 = -\frac{e^2}{2} + 1$$

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