INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #18 — 10' April 10, 2019

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Exercise 1. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Give an expression of the antiderivative F of f such that $F(3) = -1$.

$F(x) = \int_3^x f(x)\,dx - 1$ ✓

Exercise 2. Solve the following linear system using the Gaussian elimination. You will explicitly mention the elementary row operations you're performing at each step of the descent.

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$$(S)\begin{cases} -x+y+z=1 \\ 2x-y-2z=-2 \\ -3x+y+4z=-1 \end{cases} \underset{\substack{R_2 \leftarrow R_2+2R_1 \\ R_3 \leftarrow R_3-3R_1}}{\iff} \begin{cases} -x+y+z=1 \\ y=0 \\ -2y+z=-4 \end{cases} \underset{R_3 \leftrightarrow R_2}{\iff} \begin{cases} -x+z+y=1 \\ z-2y=-4 \\ y=0 \end{cases}$$ ✓

$$\underset{\text{Back substitution}}{\iff} \begin{cases} -x-4=1 \\ z=-4 \\ y=0 \end{cases} \iff \begin{cases} x=-5 \\ z=-4 \\ y=0 \end{cases}$$ ✓

What is the rank of the system (S)?

$\mathrm{rk}(S) = 3$ ✓

Exercise 3. Give an antiderivative F of the following function:

$$f : \mathbb{R}_+^* \longrightarrow \mathbb{R}$$
$$x \longmapsto \frac{x-1}{x(x^2+1)}. \quad -1$$

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$f(x) = \frac{A}{x} + \frac{Cx+D}{x^2+1}$ $Ci+D = \frac{i-1}{i} = \frac{i(i-1)}{-1} = \frac{-1-i}{-1} = 1+i$

$A = \cancel{0}\ -1$ $C=1$ $D=1$ $f(x) = \frac{x+1}{x^2+1} = \frac{1}{2}\frac{2x}{x^2+1} + \frac{1}{x^2+1}$

so ?

$F(x) = \frac{1}{2}\ln(x^2+1) + \arctan(x^2+1)$ $-\ln(x)$