INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #7 — 12'

November 21, 2018

Name ? C'est marqué au dos.

15/20

Exercise 1. Determine, if they exist, the following values (if a value doesn't exist, cross out the equal sign and write *DNE*). No justifications required.

8

$\inf([0,2]) = 0$	$\min([0,2]) = 0$
$\sup((0,2)) = 2$	$\max((0,2)) =$ DNE
$\inf(\mathbb{Z}) = -\infty$	$\min(\mathbb{Z}) =$ DNE
$\inf(\mathbb{Z}\cap[-1/2, 9/2]) = 0$ [0;3]	$\min(\mathbb{Z}\cap[-1/2,9/2]) = 0$

Exercise 2. Recall the *definition* of the following statements:

- For a function f defined in a punctured neighborhood of -4, the statement "$\lim_{-4} f = 1$" means:

2 $x_0 \in \mathbb{R}$, $\forall \varepsilon > 0, \exists \delta > 0, \forall x \in (-4-\delta; -4) \cup (-4+\delta), |f(x) - 1| < \varepsilon$

- For a function f defined in a right-sided punctured neighborhood of -4, the statement "$\lim_{-3^+} f = 1$" means: ??

1/3 ~~$x_0 \in \mathbb{R}, \forall \varepsilon > 0, \exists \delta > 0, \forall x \in$~~ Nothing !?!?

Exercise 3. Let A be a non-empty subset of $\mathbb{R}$, and let $m \in \mathbb{R}$. Recall the definition of "m is a lower bound of A."

2 m is a lower bound of $A \Leftrightarrow \forall a \in A, a \geq m$

Exercise 4. *Prove* that $\inf(4,6] = 4$.

$m < \frac{m}{2} < \frac{m+1}{2}$

2

By definition of a lower bound, $\forall x \in (4;6]$; $x > 4$ then $x \geq 4$.

So 4 is a lower bound of $(4,6]$. ✓

Now, let's show that 4 is the greatest lower bound of $(4;6]$.

- Let $m_1 \in (-\infty; 4)$ $m_1 < 4$ then m_1 cannot be the greatest lower bound of $(4;6]$ as 4 is already an upper bound of $(4;6]$ and $m_1 < 4$.
- by contradiction, let's take $m_2 \in (4;6]$ and consider that m_2 is an upper bound

then $m_2 - 1 < m_2$

$\frac{m_2 - 1}{2} < \frac{m_2}{2}$

~~Then we deduce 4 is the only ~~lowest~~ greatest lower bound of (4;6.~~