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INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #9 — 10'

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Exercise 1. Determine the value of the following limits. You're asked to write your answer in the following form:

`∀variable ∈ appropriate neighborhood,`

`expression given = expression that enables you to compute the limit` $\underset{\text{variable}\to\text{point}}{\longrightarrow}$ `value of limit` (or DNE).

a) $\lim_{x\to+\infty} \frac{x^3-1}{x^3+x\cos(x)}$.

4

$\forall x \in (42, +\infty)$

$$\frac{x^3-1}{x^3+x\cos(x)} = \frac{x^3\left(1-\frac{1}{x^3}\right)}{x^3\left(1-\frac{\cos(x)}{x^2}\right)} = \frac{1-\frac{1}{x^3}}{1-\frac{\cos(x)}{x^2}} \underset{x\to+\infty}{\longrightarrow} 1$$

→ cos is bounded hence $\lim_{x\to+\infty} -\frac{\cos x}{x^2} = ($

b) $\lim_{x\to 0} \frac{\sin(x^2)}{xe^x}$.

4

$\forall x \in \mathbb{R}^*$

$$\frac{\sin(x^2)}{xe^x} = \frac{\sin(x^2)}{x^2} \times \frac{x}{e^x} \underset{x\to 0}{\longrightarrow} 1\times 0 = 0$$

c) $\lim_{x\to 0} \frac{\ln(1+x)}{e^x-1}$.

4

$\forall x \in (-1,0)\cup(0,+\infty)$

$$\frac{\ln(1+x)}{e^x-1} = \frac{\ln(1+x)}{x} \times \frac{x}{e^x-1} \underset{x\to 0}{\longrightarrow} 1\times 1 = 1$$

d) $\lim_{x\to 0} \frac{e^{\sin(x^2)}-1}{x^2}$.

$$\frac{e^{\sin(x^2)}-1}{\sin(x^2)} \times \frac{\sin(x^2)}{x^2}$$

1

1

$\forall x \in \mathbb{R}^*$

$$\frac{e^{\sin(x^2)}-1}{x^2} = \quad ? \quad \underset{x\to 0}{\longrightarrow} 1$$

Exercise 2. Let $a \in \mathbb{Z}$, let $(u_n)_{n\geq a}$ be a sequence of real numbers and let $\ell \in \mathbb{R}$. Recall the definition of "$\lim_{n\to+\infty} u_n = \ell$."

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$\forall \varepsilon > 0, \exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N} > n_0, |U_n - \ell| < \varepsilon$