INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #15 — 12' April 15, 2022

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Exercise 1. Recall the Rank–Nullity Theorem.

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let $f : E \to F$ a linear map
$\dim E = \dim \operatorname{Ker} f + \dim \operatorname{Im} f$
$= \dim \operatorname{Ker} f + \operatorname{rk} f$

Exercise 2. Let $f\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be a linear map such that

$$f\big((1,0,1)\big) = (3,1), \qquad f\big((1,2,1)\big) = (2,2).$$

Determine the value of $f\big((-1,2,-1)\big)$.

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$f((-1,2,-1)) = (-4,0)$

Exercise 3. Let

$$f: \begin{array}{ccc} \mathbb{R}^3 & \longrightarrow & \mathbb{R}^3 \\ (x,y,z) & \longmapsto & (x+3y+3z, x-3y-3z, y+z). \end{array}$$

$\begin{cases} x+3y+3z=0 \\ x-3y-3z=0 \\ y+z=0 \end{cases} \underset{C_2+C_1}{\iff} \begin{cases} x+3y+3z=0 \\ 2x = 0 \\ y+z=0 \end{cases}$

You're given that f is a linear map.

1. Determine the kernel of f. If $\operatorname{Ker} f \neq \{(0,0,0)\}$, give a basis of $\operatorname{Ker} f$.

$\begin{cases} y=-t \\ z=t \\ x=0 \end{cases}$ $(0,-1,1)$

0

$\operatorname{Ker} f = \{u \in E \mid f(u) = 0_F\}$ $\begin{cases} x+3y+3z=0 \\ x-3y-3z=0 \\ y+z=0 \end{cases} \underset{R_2 \leftarrow R_2 - R_1}{\iff} \begin{cases} x+3y+3z=0 \\ -6y-6z=0 \\ y+z=0 \end{cases} \underset{R_2 \leftarrow R_2+6R_3}{\iff} \begin{cases} x+3y+3z=0 \\ 0=0 \\ y+z=0 \end{cases}$

2. Determine $\operatorname{Im} f$. If $\operatorname{Im} f \neq \{(0,0,0)\}$, give a basis of $\operatorname{Im} f$.

so $\begin{cases} x=\cancel{-6z}\ 0 \\ y=z \\ z=z \end{cases}$ A basis of Ker f is $(\cancel{z}(\cancel{x},-1,-1))$ 0

0

$\operatorname{Im} f = \{f(u);\ u \in E\}$. We have to determine the rank of f in the standard base (x,y,z)

$\begin{pmatrix} 1 & 1 & 0 \\ 3 & -3 & 1 \\ 3 & -3 & 1 \end{pmatrix} \iff \begin{pmatrix} 1 & 2 & 0 \\ 3 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix}$ rk 2

$\begin{pmatrix} 1 & 2 & 2 \\ 1 & -2 & -2 \\ 0 & 1 & 1 \end{pmatrix} \underset{C_2 - C_2 - C_1}{\iff} \begin{pmatrix} 1 & 0 & 2 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix}$

3. Is f injective? surjective? (justify your answer as concisely as possible)

so rk Im f = 2 basis ((1,1,0), (2,-2

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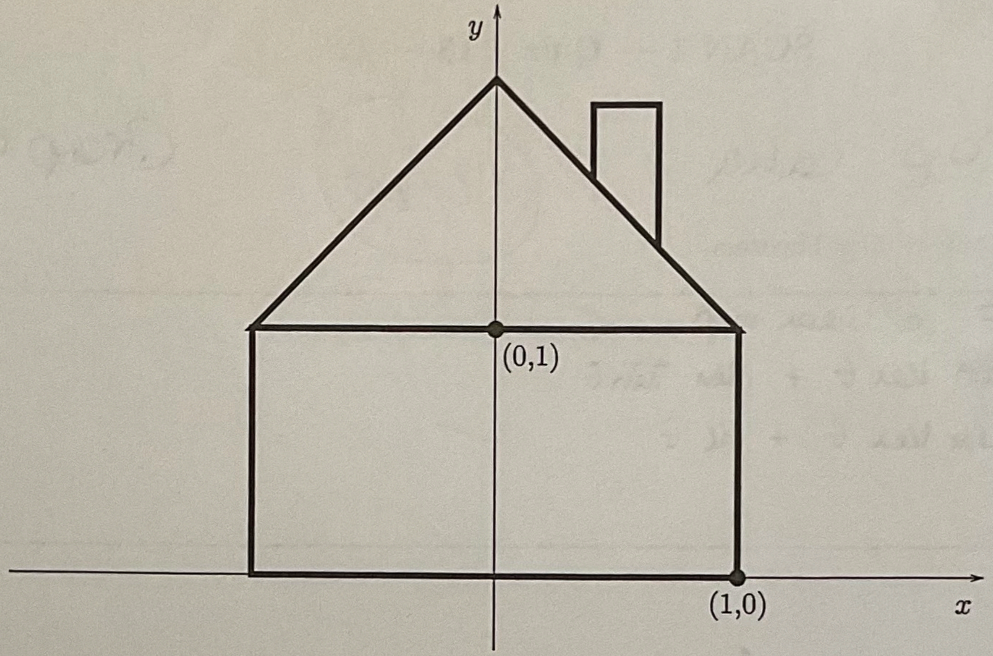
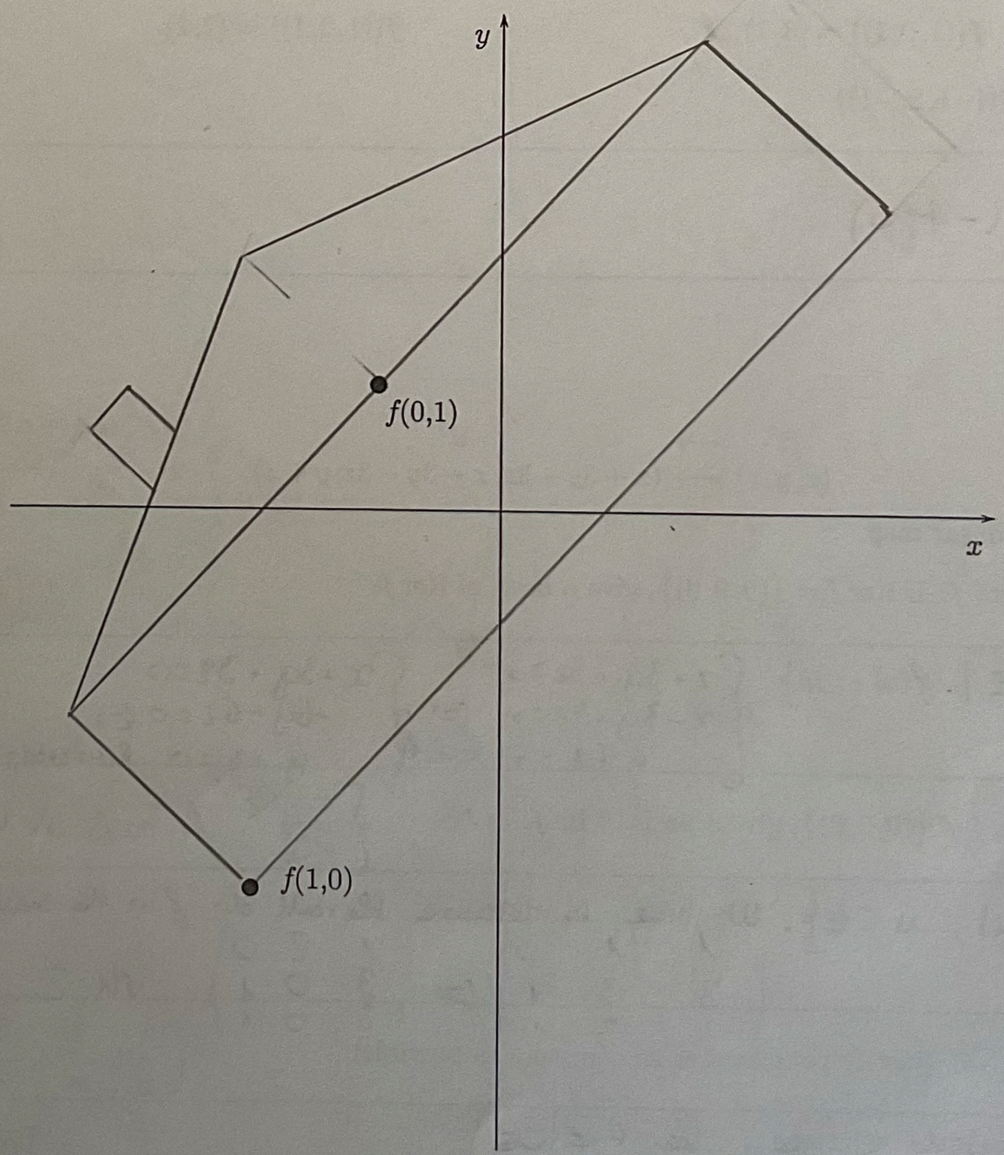
f is not injective since $\operatorname{Ker} f \neq \{0_E\}$
f is surjective iff ~~Im f = F~~

Since rk Im f ≠ rk $\mathbb{R}^3$ it is not surjective and so not injective

Exercise 4. The figures are on the back of this sheet.
Let $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the endomorphism of $\mathbb{R}^2$ such that $f(1,0)$ and $f(0,1)$ are shown in the second figure. Plot on the second figure the image by f of the house shown in the first figure.





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