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Exercise 1. Fill in the blank:

$$\checkmark \cos\left(-\frac{46\pi}{3}\right) = \cos\left(-\frac{45\pi}{3} - \frac{\pi}{3}\right) = \cos\left(-15\pi - \frac{\pi}{3}\right) = \cos\left(-\pi - \frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$$

Exercise 2. Let $n \in \mathbb{N}$. Simplify:

$$\checkmark \frac{(2n+5)!}{(2n+3)!} = \frac{(2n+3)!(2n+4)(2n+5)}{(2n+3)!} = (2n+4)(2n+5)$$

Exercise 3. Let $\theta \in \mathbb{R}$ such that $\cos(\theta) = 1/5$. Determine the value of $\cos(2\theta)$:

$$\checkmark \cos(2\theta) = 2\cos^2(\theta) - 1 = 2 \times \left(\frac{1}{5}\right)^2 - 1 = \frac{2}{25} - 1 = -\frac{23}{25}$$

Exercise 4. Let $x \in \mathbb{R}$. Fill in the blanks:

$$\checkmark \sin(x) = \frac{1}{2} \Leftrightarrow \begin{array}{l} \textcircled{a} \sin(x) = \sin\left(\frac{\pi}{6}\right) \\ \exists k \in \mathbb{Z}, x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = \pi - \frac{\pi}{6} + 2k\pi \\ \textcircled{b} \exists k \in \mathbb{Z}, x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = \frac{5\pi}{6} + 2k\pi \end{array}$$

Exercise 5. Let $u, v \in \mathbb{R}$. Fill in the blank with the addition (or subtraction) formula:

$$\checkmark \begin{array}{l} \cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v) \\ \sin(u-v) = \sin(u)\cos(v) - \sin(v)\cos(u) \end{array}$$

Exercise 6. Let $s \in \mathbb{R}$. Recall the half-angle formula:

$$\checkmark \sin^2(s) = \frac{1 - \cos(2s)}{2}$$

Exercise 7. Fill in the blanks:

$$\checkmark \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \quad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

Exercise 8. Write the following expression with the \sum symbol.

$$\checkmark 1^1 - 3^2 + 5^3 - 7^4 + 9^5 \dots - 25^{12} = \sum_{k=2}^{12} (2k-1)^k (-1)^{k+1}$$