INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

20

SCAN 1 — Quiz #2 — 10' *September 29, 2021*

Name: CARBON Clémentine

Exercise 1. Let $n \geq 2$. Fill in the blank so that the two sums are equal (by a shift of index):

$$\sum_{k=2}^{n}(2k-1)^2 = \sum_{k=0}^{n-2} (2(k+2)-1)^2 = \sum_{k=0}^{n-2} (2k+3)^2$$

4

Exercise 2. Let

$$f : \begin{array}{ccc}\mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & x^2+1\end{array} \quad \text{and} \quad g : \begin{array}{ccc}\mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & x^4\end{array} \quad \text{and} \quad h : \begin{array}{ccc}\mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & x+1.\end{array}$$

Fill in the blank:

$\forall x \in \mathbb{R}$, $(h \circ f \circ g)(x) =$ $h(f(g(x))) = h(f(x^4)) = h((x^4)^2+1)$

$= ((x^4)^2+1)+1$

$= x^8+2$

4

Exercise 3. Let

$$f : \begin{array}{ccc}\mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & x^2.\end{array}$$

Determine the following images and inverse images. If an image or inverse image doesn't exist, cross out the equal sign and write "DNE."

$f([1,3]) =$ $[1;9]$ $\quad f([-3,1]) =$ $[0;9]$

$f^{[-1]}([-1,4]) =$ $[-2;2]$ $\quad f^{[-1]}([-4,-1]) =$ $\varnothing$

4

Exercise 4. Let $n \in \mathbb{N}$ and $q \in \mathbb{C} \setminus \{1\}$. Recall the formula for the sum of the terms of a geometric progression:

$$\sum_{k=0}^{n} q^k = \frac{1-q^{n+1}}{1-q}$$

4

Exercise 5. Let $n \in \mathbb{N}$. Simplify as much as possible:

$$\frac{\binom{2n+1}{n}}{\binom{2n}{n}} = \frac{\frac{(2n+1)!}{n!\,(2n+1-n)!}}{\frac{(2n)!}{n!\,(2n-n)!}} = \frac{\frac{(2n+1)!}{n!\,(n+1)!}}{\frac{(2n)!}{n!\,n!}}$$

$$= \frac{(2n)!\,(2n+1)}{(n+1)!} \times \frac{n!}{(2n)!}$$

$$= \frac{(2n+1) \times n!}{n!\,(n+1)} = \frac{2n+1}{n+1}$$

4