INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1 — Quiz #3 — 12'

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Exercise 1. Let A be a non-empty set, let $f : A \to \mathbb{R}$ and let $Q \in \mathbb{R}$. Recall the definition of

"Q is a lower bound of f."

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Q is a lower bound of $f \iff \forall x \in A,\ f(x) \geqslant Q$

Exercise 2. Let $p, b \in \mathbb{C}$ and $k \in \mathbb{N}$. Recall the Binomial Theorem (and mind the notations!):

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$$(p+b)^k = \sum_{\ell=0}^{k} \binom{k}{\ell} p^{\ell} b^{k-\ell}$$

Exercise 3. Let $x \in \mathbb{R}$. Fill in the blank (only give the final answer, no justifications required):

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$$\cos(3x) = \frac{1}{2} \iff x = \frac{\pi}{9} + \frac{2k\pi}{3} \text{ or } x = \frac{-\pi}{9} + \frac{2k\pi}{3}$$

Exercise 4. Let $a, b \in \mathbb{R}$. Recall the product formula:

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$$\cos(a)\cos(b) = \frac{1}{2}\left(\cos(a+b) + \cos(a-b)\right)$$

Exercise 5. Let $A \subset \mathbb{R}$, let $f : A \to \mathbb{R}$, and let $B \subset A$. Recall the definition of "f is decreasing on B."

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f is decreasing on $B \iff \forall x, y \in B\ (x < y \Rightarrow f(x) > f(y))$

Exercise 6. Let A be a non-empty subset of $\mathbb{R}$ that is symmetric with respect to 0, and let $f : A \to \mathbb{R}$ and $g : A \to \mathbb{R}$ be two *even* functions. Prove that the function $f + g : A \to \mathbb{R}$ is even.

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Let $x \in \cancel{\mathbb{R}}\ A$ since $f: A \to \mathbb{R}$ and $g: A \to \mathbb{R}$

$f+g: A \to \mathbb{R}$

since A is symmetric with respect to 0, $-x \in A$

$(f+g)(-x) = f(-x) + g(-x)$

since f and g are even

$(f+g)(-x) = f(x) + g(x) = (f+g)(x)$

hence $f+g : A \to \mathbb{R}$ is even.