MTES – Correction of the exam #2

Exercise 1 (3 points)

1a	$dh = \frac{\partial h}{\partial R}dR + \frac{\partial h}{\partial V}dV = \frac{-2V}{\pi R^3}dR + \frac{1}{\pi R^2}dV$ So $\delta h = \frac{-2V}{\pi R^3}\delta R + \frac{1}{\pi R^2}\delta V$ And $\frac{\delta h}{h} = -2\frac{\delta R}{R} + \frac{\delta V}{V}$	0.25 (formula) 0.5 ($\frac{\partial h}{\partial R}$) + 0.25 ($\frac{\partial h}{\partial V}$) 0.5
1b	Numerical application: $\frac{\delta h}{h} = -2.10^{-3} + 6.10^{-3} = 0.4\ \%$	0.25 (even with wrong unit)
2a	$\frac{\Delta h}{h} = 2\frac{\Delta R}{R} + \frac{\Delta V}{V}$	0.5 (0.25 in case of wrong expression but absolute values)
2b	Numerical application: $\frac{\Delta h}{h} = 2.10^{-3} + 10^{-2} = 1.2\ \%$	0.25 (even with wrong unit)
2c	$h = (3.1831\ \pm 0.039)\ m$	0.5 (correct writing) 0 if no unit, or if not coherent with the value of $\frac{\Delta h}{h}$

Exercise 2 (2 points)

1	$df' = \frac{\partial f'}{\partial L}dL + \frac{\partial f'}{\partial l}dl$ $\frac{\partial f'}{\partial L} = \frac{L^2+l^2}{4L^2}$ and $\frac{\partial f'}{\partial l} = \frac{-l}{2L}$ Since L and l are both positive, we get $\Delta f' = \frac{L^2+l^2}{4L^2}.\Delta L + \frac{l}{2L}.\Delta l$ With $\Delta L = \Delta l = \varepsilon$, the expression becomes: $\Delta f' = \frac{(L+l)^2}{4L^2}.\varepsilon$	0.25 0.5 + 0.25 0.25 absolute values + 0.25 justification 0.5

Exercise 3 (4 points)

1	ω is not closed since $\frac{\partial(\frac{-x}{y})}{\partial x} \neq \frac{\partial(\frac{y}{2}+1)}{\partial y}$	0.25 if $\frac{\partial(\frac{-x}{y})}{\partial x}$ and $\frac{\partial(\frac{y}{2}+1)}{\partial y}$ are correct
2a	Ω is closed iff $\frac{\partial(\phi(y).\frac{-x}{y})}{\partial x} = \frac{\partial(\phi(y).(\frac{y}{2}+1))}{\partial y}$ i.e. $\frac{-\phi(y)}{y} = \phi'(y).\left(\frac{y}{2}+1\right) + \frac{\phi(y)}{2}$ or $\phi'(y).y = -\phi(y)$	0.25 0.5 (0.25 for $\frac{\partial(\phi(y).\frac{-x}{y})}{\partial x}$ and 0.25 for $\frac{\partial(\phi(y).(\frac{y}{2}+1))}{\partial y}$) 0.5

2b	The relation $\phi'(y).y = -\phi(y)$ is valid for $\phi(y) = \frac{K}{y}$, so Ω is closed	0.5 (OK when checked Ω was closed)
2c	By integration, $f(x,y) = \frac{x}{2} + \frac{x}{y} + constant$	0.5 integration wrt x, with the introduction of a function g(y) 0.5 derivation 0.5 integration wrt y 0.5 final expression with constant

Exercise 4 (7 points)

1	Graph with a symmetry of revolution around the z-axis	0.5 + 0.5
2	$x(t) = r.cos\theta = e^{-2t}\cos(2\pi t)$ $y(t) = r.sin\theta = e^{-2t}\sin(2\pi t)$ $z(t) = e^{-t}$	0.25 0.25 0.25
3	$(x^2+y^2)^{1/4} = e^{-t} = z$	0.25
4	$\overrightarrow{OM}(t) = e^{-2t}\cos(2\pi t)\overrightarrow{e_X} + e^{-2t}\sin(2\pi t)\overrightarrow{e_Y} + e^{-t}\overrightarrow{e_Z}$	0.5
5	$\vec{v} = \frac{d\overrightarrow{OM}}{dt}$ $\vec{v} = [-2e^{-2t}\cos(2\pi t) - 2\pi e^{-2t}\sin(2\pi t)]\overrightarrow{e_X} + [-2e^{-2t}\sin(2\pi t) + 2\pi e^{-2t}\cos(2\pi t)]\overrightarrow{e_Y} - e^{-t}\overrightarrow{e_Z}$	0.25 0.25 0.25
6	$\overrightarrow{e_r}(t) = cos\theta\overrightarrow{e_X} + sin\theta\overrightarrow{e_Y} = \cos(2\pi t)\overrightarrow{e_X} + \sin(2\pi t)\overrightarrow{e_Y}$ $\overrightarrow{e_\theta}(t) = -sin\theta\overrightarrow{e_X} + cos\theta\overrightarrow{e_Y} = -\sin(2\pi t)\overrightarrow{e_X} + \cos(2\pi t)\overrightarrow{e_Y}$ $\overrightarrow{e_z}(t) = \overrightarrow{e_Z}$	0.25+0.25 0.25+0.25 0.25
7	$\overrightarrow{OM}(t) = r\overrightarrow{e_r} + z\overrightarrow{e_Z} = e^{-2t}\overrightarrow{e_r} + e^{-t}\overrightarrow{e_Z}$	0.25 + 0.25
8	$\vec{v} = \frac{d\overrightarrow{OM}}{dt} = \dot{r}\overrightarrow{e_r} + r\dot{\overrightarrow{e_r}} + \dot{z}\overrightarrow{e_Z} + z\dot{\overrightarrow{e_Z}}$ With $\dot{\overrightarrow{e_r}} = \dot{\theta}\overrightarrow{e_\theta} = 2\pi\overrightarrow{e_\theta}$ And $\dot{\overrightarrow{e_Z}} = \vec{0}$ So $\vec{v} = -2e^{-2t}\overrightarrow{e_r} + 2\pi e^{-2t}\overrightarrow{e_\theta} - e^{-t}\overrightarrow{e_Z}$	0.5 0.25 0.25 0.5
9	path	0.5

Exercise 5 (4 points)

1	Erstfeld: $\begin{cases} r_E = 6379.665\ km \\ \theta_E = 43.17473\,° = 0.75354\ rad \\ \varphi_E = 8.645389° = 0.15089\ rad \end{cases}$ Bodio: $\begin{cases} r_B = 6378.321\ km \\ \theta_B = 43.621687\,° = 0.76134\ rad \\ \varphi_B = 8.910725° = 0.15552\ rad \end{cases}$	0.25 (values of r) 0.5 (θ in ° or rad) 0.25 (φ in ° or rad) ~~0 if no unit~~
2	Conversion to cartesian coordinates : $\begin{cases} x = rsin\theta cos\varphi \\ y = rsin\theta sin\varphi \\ z = rcos\theta \end{cases}$	0.25 0.25 0.25

	So we get the cartesian coordinates of each place: Erstfeld: $\begin{cases} x_E = 4315.531\ km \\ y_E = 656.160\ km \\ z_E = 4652.501\ km \end{cases}$ Bodio: $\begin{cases} x_B = 4347.254\ km \\ y_B = 681.596\ km \\ z_B = 4617.335\ km \end{cases}$	6*0.25 ~~(0 if no unit)~~
3	$d = \sqrt{(x_E - x_B)^2 + (y_E - y_B)^2 + (z_E - z_B)^2} = 53.758\ km$	0.25 + 0.5 (0 if no unit)