IEFS Correction Grading scale – Jan. 20, 2023

Exercise 1 (4 points)

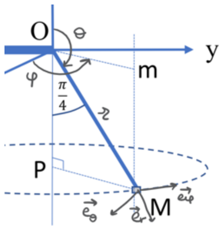
1.	ω is closed if and only if $\dfrac{\partial Q}{\partial x} = \dfrac{\partial P}{\partial y}$. Here, $\dfrac{\partial Q}{\partial x} = 0$ et $\dfrac{\partial P}{\partial y} = 2y$ so ω is not closed.	0.5
2. (a)	ω_1 is closed $\iff \dfrac{\partial(\varphi(x)Q)}{\partial x} = \dfrac{\partial(\varphi(x)P)}{\partial y} \iff \varphi'(x)2y = \varphi(x)2y \iff \varphi'(x) = \varphi(x)$, $\forall$x$\in \mathbb{R}$. With the initial condition give, $\varphi(x) = \mathrm{e}^x$.	1
(b)	$\omega_1 = \mathrm{e}^x\omega(x)$ is closed on $\mathbb{R}^2$ so it is exact.	0.5
(c)	$\mathrm{d}f = \dfrac{\partial f}{\partial x}\mathrm{d}x + \dfrac{\partial f}{\partial y}\mathrm{d}y$ hence $df = \omega_1$ iff $\begin{cases} \dfrac{\partial f}{\partial x} = \mathrm{e}^x(x^2+y^2+2x) \quad (1) \\ \dfrac{\partial f}{\partial y} = \mathrm{e}^x 2y \quad (2) \end{cases}$ By anti-derivation of (2) with respect to y : $f(x,y) = \mathrm{e}^x y^2 + g(x)$. By deriving f wrt x : $\dfrac{\partial f}{\partial x} = \mathrm{e}^x y^2 + g'(x) = \mathrm{e}^x(x^2+y^2+2x)$ so and by identification with (1), $g'(x) = x^2\mathrm{e}^x + 2x\mathrm{e}^x$, hence $g(x) = x^2\mathrm{e}^x + C$ where $C \in \mathbb{R}$ is a constant. Conclusion : $\boxed{f(x,y) = \mathrm{e}^x(x^2+y^2) + C,\ C \in \mathbb{R}}$. *(the calculations being simple, be demanding on this question)*	2

Exercise 2 (5 points)

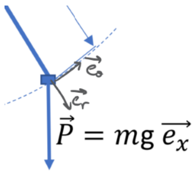
1.	a. $f'(a,a') = \frac{a.a'}{a-a'}$ b. $\mathrm{d}f = \dfrac{-a'^2}{(a-a')^2}\mathrm{d}a + \dfrac{a^2}{(a-a')^2}\mathrm{d}a'$ for $a \neq a'$. d. Numerical application : $f' = (-20.00 \pm 0.60)\ cm$ c. $\Delta f = \dfrac{a'^2}{(a-a')^2}\Delta a + \dfrac{a^2}{(a-a')^2}\Delta a'$	0.5 1 0.5 0.5
2.	a. $a' = 20\ cm$ b. $\delta a' = \dfrac{f'^2}{(a+f')^2}\delta a + \dfrac{a^2}{(a+f')^2}\delta f'$ Here $\delta f' = 0$ so $\delta a' = \dfrac{f'^2}{(a+f')^2}\delta a$ c. $\delta a' = -1\ cm$	0.5 1 0.5 0.5

Exercise 3

Part A (3 points)

1.		0.5 (r,θ,φ) 0.5 (frame)
2.	$\overrightarrow{OM} = (r\sin\theta\cos\varphi)\vec{e}_x + (r\sin\theta\sin\varphi)\vec{e}_y + (r\cos\theta)\vec{e}_z$ $\overrightarrow{OM} = r\vec{e}_r$.	0.5 0.5
3.	Parametric equation of the trajectory : $\begin{cases} \text{r=L} \\ \theta = \frac{3\pi}{4} \\ \varphi = \omega t \\ \text{t} \in \text{R} \end{cases}$	0.5 $(r,\ \theta)$ 0.5 (φ)

Part B (5 points)

1.	 	0.5
2.	$\vec{e}_r = \cos\theta\vec{e}_x + \sin\theta\vec{e}_y$ $\vec{e}_\theta = -\sin\theta\vec{e}_x + \cos\theta\vec{e}_y$ $\frac{d\vec{e}_r}{dt} = \frac{d}{dt}(\cos\theta\vec{e}_x + \sin\theta\vec{e}_y) = -\theta'\sin\theta\vec{e}_x + \theta'\cos\theta\vec{e}_y = \theta'\vec{e}_\theta$	0.25 0.25 0.5
3. (a)	$\overrightarrow{OM} = r\vec{e}_r = L\vec{e}_r$. L being constant : $\vec{v} = \frac{d\overrightarrow{OM}}{dt} = L\frac{d\theta}{dt}\vec{e}_\theta$.	0.5
(b)	$\vec{a} = \frac{d\vec{v}}{dt} = L\frac{d^2\theta}{dt^2}\vec{e}_\theta + L\frac{d\theta}{dt}\frac{d\vec{e}_\theta}{dt} = = L\frac{d^2\theta}{dt^2}\vec{e}_\theta - L\left(\frac{d\theta}{dt}\right)^2\vec{e}_r$.	1
4. (a)	For θ close to 0 : $\sin(0+\theta) \simeq \sin(0) + \sin'(0)\theta \simeq \theta$.	1
(b)	homogeneous 2nd order differential equation : $\theta'' + \frac{g}{L}\theta = 0$ Characteristic equation $x^2 + \frac{g}{L} = 0 \iff x = \pm i\sqrt{\frac{g}{L}}$ (**0.5pt**). Solutions : $\theta(t) = c_1\cos(\sqrt{\frac{g}{L}}t) + c_2\sin(\sqrt{\frac{g}{L}}t)$ (ou $\theta(t) = k_1\cos(\sqrt{\frac{g}{L}}t + k_2)$, $c_1, c_2, k_1, k_2 \in \mathbb{R}$) (**0.5pt**).	1

Part C (3 points)

1.	First order equation : Particular solution : $v_p = \frac{mg}{\lambda}$. Characteristic equation of the complementary homogeneous equation : $mr + \lambda = 0$, of root $r = -\frac{\lambda}{m}$ Hence the general solution of the complementary homogeneous equation is : $v_h = A\exp(-\frac{\lambda}{m}t)$ The solution of the equation is therefore : $v = v_p + v_h = \frac{mg}{\lambda} + Ae^{-\frac{\lambda}{m}t}$ initial conditions : $v(0) = 0$ hence $A = -\frac{mg}{\lambda}$ and $v = \frac{mg}{\lambda}(1 - e^{-\frac{\lambda}{m}t})$	1

2.	**import numpy as np** a = 0 b = 10 m = 1 mu = 1 **g = 9.81**
	n = **1000** # number of subintervals
	t = np.ones(n+1) * 0 # initialisation of the array t of size n
	v = np.ones(n+1) * 0 # of v with the initial condition
	h = (b-a)/n # definition of the step
	for k in range(1,n+1) :
	t[k]=t[k-1]+h
	v[k]=v[k-1]+**h*(g-v[k-1]*mu/m)**
	print(v)
		+0.5 per missing element (in bold)