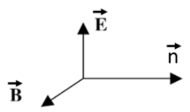
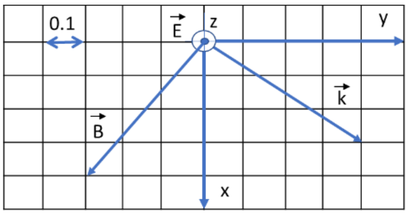
Grading scale IE1 OMSI – November, 18 (/21 + 2pt de bonus)

Exercise 1 (Vectors)			**Total : 10.5 points**
1 . (a)	Conditions on the dot products → $\vec{E}$ *and* $\vec{B}$ *perpendicular to* $\vec{n}$, Condition son the cross products → $\vec{n}$, $\vec{E}$, $\vec{B}$ for a direct basis		0.5 (perpendicularity) 0.5 (direct basis) 0.5 (scheme)
1. (b)	$\|\vec{B}\| = \frac{\|\vec{E}\|}{V}$		0.5
2.	$\vec{n} = \frac{\vec{k}}{\|\vec{k}\|}$ $\|\vec{k}\| = \sqrt{(0.3)^2 + (0.4)^2} = \frac{1}{2}$ $\vec{n} = \frac{\vec{k}}{\|\vec{k}\|} = 0.6\,\vec{e_x} + 0.8\,\vec{e_y}$		0.5 (collinearity with $\vec{k}$) 0.5 ($\|\vec{k}\|$) 0.5 (components of $\vec{n}$)
3.	$\vec{B} = \frac{1}{V}\frac{\vec{k}}{\|\vec{k}\|} \wedge \vec{E} = \frac{1}{V}\begin{pmatrix}0.6\\0.8\\0\end{pmatrix} \wedge \begin{pmatrix}0\\0\\E_z\end{pmatrix}$ $= \frac{E_z}{V}\begin{pmatrix}0.8\\-0.6\\0\end{pmatrix} = \frac{0.8\,E_z}{V}\,\vec{e_x} - \frac{0.6\,E_z}{V}\,\vec{e_y}$		0.5 1
4.	$\vec{B}.\vec{e_x} = \frac{0.8E_z}{V} = \|\vec{B}\|.\|\vec{e_x}\|.\cos(\vec{B},\vec{e_x})$ $\|\vec{B}\| = \frac{\|\vec{E}\|}{V} = \frac{E_z}{V}$ $\cos(\vec{B},\vec{e_x}) = 0.8$ $\vec{B}.\vec{e_z} = 0$ $\cos(\vec{B},\vec{e_z}) = 0$ i.e. $\vec{B} \perp \vec{e_z}$		0.5 0.5 0.5
5.			0.5
6.	$\overrightarrow{AM}\,.\,\vec{n} = 0$ with A (2,1,0) and $\vec{n} = \begin{pmatrix}0.6\\0.8\\0\end{pmatrix}$ $(x-2)\,0.6 + (y-1)\,0.8 = 0$ $0.6\,x + 0.8\,y - 2 = 0$		0.5 (explanation) 0.5 (equation) Give the points if the students give the parametric equation

7. (a)	$P = S\left(\vec{E} \wedge \frac{1}{\mu_0}\vec{B}\right).\left(u_x\vec{e_x} + u_y\,\vec{e_y}\right)$ We can take, for instance, $\left(u_x = n_y\ and\ u_y = -n_x\right)$ $\begin{cases} u_y = \frac{0.3}{0.4}u_x \\ {u_x}^2 + {u_y}^2 = 1 \end{cases}$ $\begin{cases} u_x = \frac{0.16}{0.25} \\ u_y = \left(\frac{0.3}{0.4}\right)\left(\frac{0.16}{0.25}\right) \end{cases}$	0.5 1 (system) 0.5 (result)
7. (b)	$P = \frac{SE_z^2}{5\mu_0 V}4$	0.5 (0.25 if wrong sign)

Exercise 3 (Complex numbers)		**Total : 5 points +1 bonus point**
1.	$z_1 = 2e^{i\frac{\pi}{4}}$ (Modulus + Argument) $z_1 = 2e^{i\frac{3\pi}{2}}$ (Modulus + Argument)	0.5 0.5
2.	$z = \frac{z_1 * z_2}{z_1 + z_2} = \frac{4 * (\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2})}{(\sqrt{2} + i * (\sqrt{2} - 2))} = \frac{2 * \sqrt{2} * (1 - i)}{\sqrt{2} * (1 + i(1 - \sqrt{2}))}$ $= \frac{2 * (1 - i)}{1 + i(1 - \sqrt{2})} = \frac{\sqrt{2} + i * (\sqrt{2} - 2)}{2 - \sqrt{2}}$ Real part : $\frac{\sqrt{2}}{2-\sqrt{2}}$ Imaginary part : = - 1 (not -i)	1.5 (calculation of z) 0.25 0.25
3.	$\lvert z\rvert = \sqrt{\left(\frac{\sqrt{2}}{2-\sqrt{2}}\right)^2 + 1} = \sqrt{\frac{4-2\sqrt{2}}{3-2\sqrt{2}}} = \frac{2}{\sqrt{2-\sqrt{2}}}$	0.5 (coherently with Q.2)
4.	$z = \frac{z_1 * z_2}{z_1 + z_2} = \frac{4 * e^{i\frac{7\pi}{4}}}{2 * \left(e^{i\frac{\pi}{4}} + e^{i\frac{3\pi}{2}}\right)} = \frac{4 * e^{i\frac{7\pi}{4}}}{2 * \left(e^{i\frac{7\pi}{8}}\right) * \left(e^{-i\frac{5\pi}{8}} + e^{i\frac{5\pi}{8}}\right)}$ $z = \frac{4 * e^{i\frac{7\pi}{4}}}{4 * \left(e^{i\frac{7\pi}{8}}\right) * \cos\frac{5\pi}{8}} = \frac{e^{i\frac{7\pi}{8}}}{\cos\frac{5\pi}{8}} = \frac{e^{-i\frac{\pi}{8}}}{\cos\frac{3\pi}{8}}$	1.5
5.	$z = \frac{\sqrt{2} + i * (\sqrt{2} - 2)}{2 - \sqrt{2}} = \frac{e^{-i\frac{\pi}{8}}}{\cos\frac{3\pi}{8}}$ So $\lvert z\rvert = \left\lvert\frac{\sqrt{2}+i*(\sqrt{2}-2)}{2-\sqrt{2}}\right\rvert = \frac{1}{\left\lvert\cos\frac{3\pi}{8}\right\rvert}$ As $\cos\frac{3\pi}{8} > 0$, we get $\cos\frac{3\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2}$	1 bonus point (coherently with Q. 3).

Exercise 2 (Parametric curves)		Total : 5.5 points + 1 bonus point
1 .	$r(t) = a;\ \theta(t) = \omega t;\ \ z(t)\ =\ -bt^2 + ct$	0.5 +0.5 + 0.5
2.	In the xy-plane : cercle of center O and radius a=2m z(t) is a parabola starting at z=0 (for t=0) and ending at z=0 for t=2, with a maximum at z=1 for t=1	0.5 0.5 (ok if only one segment along the z axis is shown)
3.	$\frac{d\overrightarrow{OM}}{dt} = -a\omega sin(\omega t)\overrightarrow{e_x} + a\omega cos(\omega t)\overrightarrow{e_y} + (-2bt + c)\overrightarrow{e_z}$ $\left\Vert\frac{d\overrightarrow{OM}}{dt}\right\Vert = \sqrt{a^2\omega^2 + (-2bt + c)^2}$ For t=0.5 , $\left\Vert\frac{d\overrightarrow{OM}}{dt}\right\Vert = \sqrt{a^2\omega^2 + (-2bt + c)^2} = \sqrt{16\pi^2 + 1}$ At t= 0.5 the point goes upwards as the component of the velocity along $\overrightarrow{e_z}$ is positive.	1 1 0.5 0.5
4.	$\Vert\overrightarrow{OM}\Vert = \sqrt{a^2 + (-bt^2 + ct)^2}$ Maximum value for $\frac{d}{dt}\Vert\overrightarrow{OM}\Vert = \frac{(-2bt+c)(-bt^2+ct)}{\sqrt{a^2+(-bt^2+ct)^2}} = 0$ $\frac{d}{dt}\Vert\overrightarrow{OM}\Vert = 0$ for t=0 (min value), t=1 (max value) and t=2 (min value) $\Vert\overrightarrow{OM}\Vert_{max} = \sqrt{a^2 + (c - b)^2}$	1 bonus point