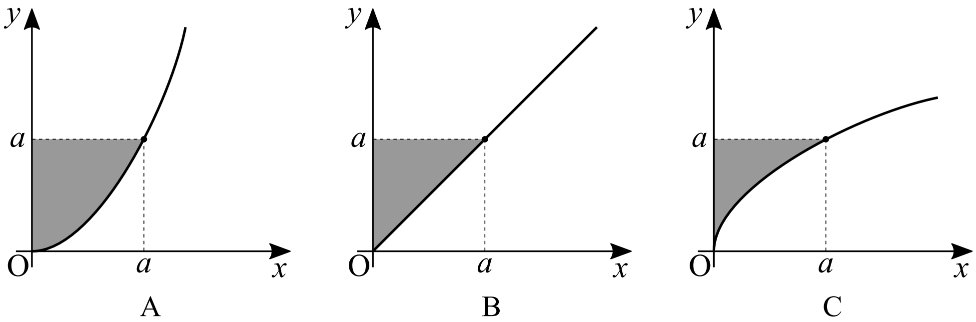
Scan First
School Year 2021-2022

MTES - S2 – Written Exam 1 (IE1)

6th of May 2022 **Total time: 1h30**

Calculators and documents are NOT authorized. Scoring is given as an indication only.

Exercise 1 (~4 pts)
We aim to calculate the shaded areas in the following four configurations (A, B and C), using a double integral.

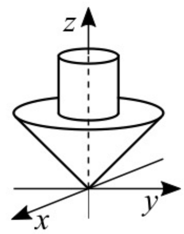


1. Associate each one of the above configurations (A to C) to one of the following integrals (normal in x):

$$I = \int_{x=0}^{a}\int_{y=\sqrt{ax}}^{a} \mathrm{d}y\,\mathrm{d}x \quad J = \int_{x=0}^{a}\int_{y=\frac{x^2}{a}}^{a} \mathrm{d}y\,\mathrm{d}x \quad K = \int_{x=0}^{a}\int_{y=x}^{a} \mathrm{d}y\,\mathrm{d}x$$

2. Rewrite integrals I, J and K as I', J' and K' this time as normal in y
3. Calculate two integrals of your choice.

Exercise 2 (~ 6 pts)
A spinner is constructed from a cone attached to a cylinder, both constituted of the same homogeneous material with density $\rho = 1$ g/cm^3, as shown in the figure to the right. The cylinder C has a radius $R = 1{,}0$ cm and height $H = 1{,}0$ cm. The cone Δ has radius $R' = 4{,}0$ cm and height $H' = 3{,}0$ cm.
We remind that the volume of a cone of radius R and height H is given by $V = \frac{\pi R^2 H}{3}$
Note: for the questions below, the results can be given as a fraction, where applicable.



1. Calculate the masses M_C and M_Δ of both cone and cylinder.
2. Determine the position of the center of inertia G_C of the cylinder.
3. Determine the position of the center of inertia G_Δ of the cone.
4. Deduce the position of the center of inertia G of the spinner.

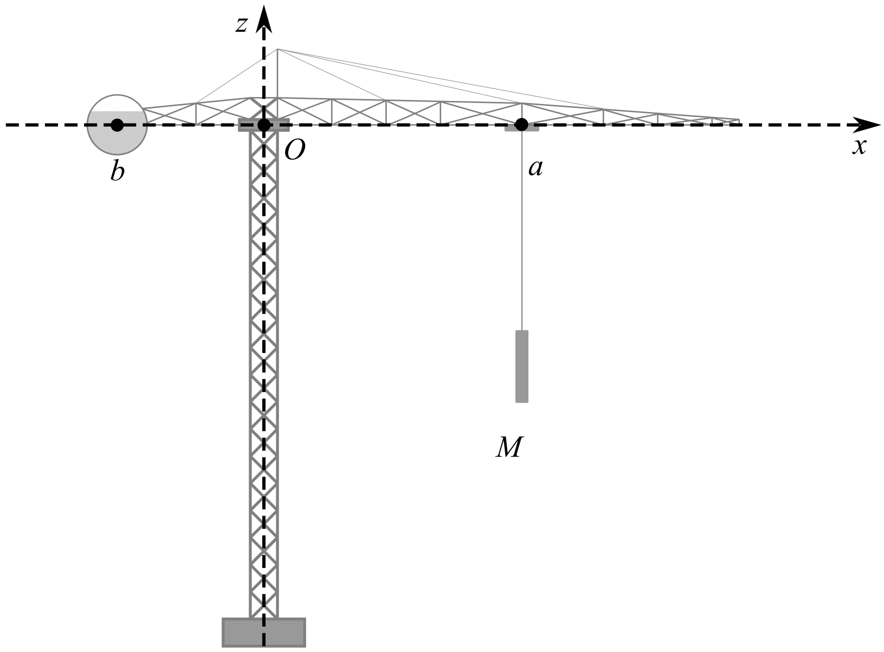
Exercise 3 (~ 6 pts)
C is a cylinder of axis (Oz), radius a and situated between the planes $z = -2a$ and $z = 2a$. Its volumetric density is given by $\rho(r, \theta, z) = k\frac{a^2+r^2}{4a+z}$ is given in cylindrical coordinates.

1. Express its mass.
2. Find a parametrization of C in Cartesian coordinates,
3. Express the distance between a point $M(x, y, z)$ and the axis (Oy).

4. Give the integral that allows to calculate the moment of inertia with respect to the axis (Oy) in Cartesian coordinates. (The solution of this integral is not required)

Exercise 4 (~ 4 pts)

*An engineer is designing a new kind of tower crane that, instead of using concrete bars as a counterweight, uses a **spherical tank of radius R**, placed at a distance b from the axis of rotation (Oz), and that can be filled with a liquid of uniform volumetric density ρ. The design of the crane is as shown in the figure below.*



The liquid can be pumped into the sphere to fill it up (making it heavier) and out of the sphere (making it lighter), to adjust the counterweight. The mass of the empty spherical tank is considered as negligible.

The crane will carry an object of mass M, supported by a cable attached to a sliding mechanism, that can slide along the x-axis. We consider the crane arm, the cable and the sliding mechanism to have negligible mass. The position of the sliding mechanism is identified by its x-coordinate a.

The engineer ultimately wants to program the computer that will pump the right quantity of liquid into the sphere as the transported object slides along the x-axis, while maintaining the balance of the crane.

Note: The first 2 questions below are independent and can be treated separately.

1. Give the expression of the mass M_S of the filled spherical tank so that the center of mass of the system {filled spherical tank and transported object} is on the z-axis.
2. Considering the symmetries of the liquid inside the spherical tank, what system of coordinates should you use to calculate the mass M_S? Find the expression of the volume of the liquid inside the reservoir as function of height $h \in [0,2R]$ of the liquid inside.
3. Give the relation between the height h, a, b, M, ρ and R.