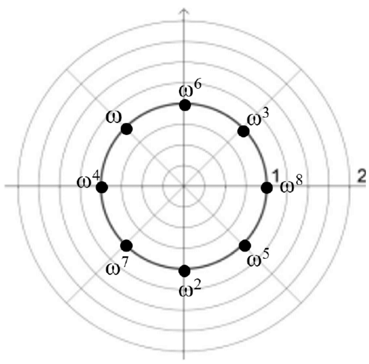
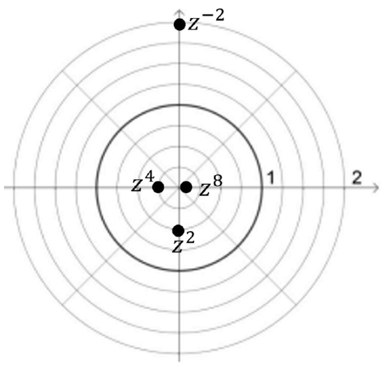
Correction of the 1st MONTY exam – Semester 1
November 22th, 2024

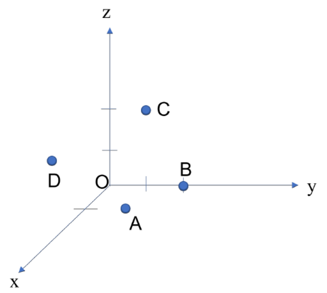
Exercise 1: Quadratic equations (3 points)

1. $z^2 = 1 + i = \sqrt{2}e^{i\pi/4}$	0.5
$z = \pm 2^{1/4}e^{i\pi}/8$	0.5
The solutions are $S = \{2^{1/4}e^{i\pi}/8; 2^{1/4}e^{i9\pi}/8\}$	0.5
2. $\Delta = 3 + 4i$ Let's find $a, b \in \mathbb{R}$ so that $\delta = a + ib$ and $\delta^2 = \Delta$: $\begin{cases} a^2 - b^2 = 3 \\ 2ab = 4 \\ a^2 + b^2 = 5 \end{cases} \Leftrightarrow \begin{cases} a = 2 \quad \text{and} \quad b = 1 \\ \text{or} \\ a = -2 \quad \text{and} \quad b = -1 \end{cases}$	0.5
We choose $\delta = 2 + i$	0.5
The solutions are $S = \{\frac{-\sqrt{3}}{2} + 1 + \frac{1}{2}i \ ; \ -\frac{\sqrt{3}}{2} - 1 - \frac{1}{2}i\}$	0.5

Exercise 2: Powers of a complex number (3 points)

1. $\omega = \frac{-\sqrt{2}}{2} + i\frac{\sqrt{2}}{2} = cos(\frac{3\pi}{4}) + sin(\frac{3\pi}{4}) = e^{\frac{3i\pi}{4}}$	0.5
 2.	1
3. $\omega^{2024} = e^{2024*3i\pi/4} = e^{1518i\pi} = 1$	0.5
4. $z^2 = \frac{-i}{2}$; $z^4 = \frac{-1}{4}$; $z^8 = \frac{1}{16}$; $z^{-2} = \frac{2}{\omega^2} = 2i$ 	1

Exercise 3 - Vectors (6 points)

 1.	0.5
2. $\vec{u} = \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{\lVert \overrightarrow{AB} \times \overrightarrow{AC} \rVert}$ $\overrightarrow{AB} \times \overrightarrow{AC} = (2\ ;\ 2\ ;\ 1)$ $\lVert \overrightarrow{AB} \times \overrightarrow{AC} \rVert = 3$ So $\vec{u} = (\frac{2}{3}\ ;\ \frac{2}{3}\ ;\ \frac{1}{3})$	0.25 0.25 0.25 0.25
3. $\overrightarrow{AH}$ is the orthogonal projection of $\overrightarrow{AD}$ onto the line directed by $\vec{u}$ So $\overrightarrow{AH} = (\overrightarrow{AD} \cdot \vec{u})\vec{u} = (\frac{4}{9}\ ;\ \frac{4}{9}\ ;\ \frac{2}{9})$	0.5 0.25
4. $\overrightarrow{AO} \cdot \vec{u} = -\frac{4}{3} < 0$ so $cos(\overrightarrow{AO}, \vec{u}) < 0$ $\overrightarrow{AD} \cdot \vec{u} = \frac{2}{3} > 0$ so $cos(\overrightarrow{AD}, \vec{u}) > 0$ Hence O and D are not on the same side of the plane (ABC)	0.25 0.25 0.25
5. Let θ be the angle $(\overrightarrow{AD}, \vec{u})$. $cos(\theta) = \frac{\overrightarrow{AD} \cdot \vec{u}}{\lVert \overrightarrow{AD} \rVert \lVert \vec{u} \rVert} = \frac{2}{3\sqrt{6}}$ Hence $\theta = cos^{-1}(\frac{2}{3\sqrt{6}})$	0.5 0.5
6. M is the center of mass of (A, a) and (B, b) iff $(a+b)\overrightarrow{OM} = a\overrightarrow{OA} + b\overrightarrow{OB}$ $\begin{cases} 2(a+b) = a \\ 0 = a + 2b \end{cases} \iff a + 2b = 0$ there are many solutions so M can be the center of mass of (A,a) and (B,b).	0.5 0.5
7. Different possibilities accepted here, among which: G is the center of mass of (I,2) and (J,2) with I the middle of AB and J the middle of CD. Hence G is the middle of IJ. Or: G is the center of mass of (I,3) and (D,1) with I at the intersection of the medians of the triangle (ABC). Hence G is on the line ID with $IG = \frac{1}{4}ID$ Graph	0.5 0.5

Exercise 4 - Differential calculus (4 points)

1. $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$	0.25
$\frac{\partial f}{\partial x}(x,y,z) = -sin(x+y^2)e^{-z}$	0.25
$\frac{\partial f}{\partial y}(x,y,z) = -2ysin(x+y^2)e^{-z}$	0.25
$\frac{\partial f}{\partial z}(x,y,z) = -cos(x+y^2)e^{-z}$	0.25
so $df = -sin(x+y^2)e^{-z}dx - 2ysin(x+y^2)e^{-z}dy - cos(x+y^2)e^{-z}dz$	0.25
2. $\omega = Pdx + Qdy$ To be closed, we need to have $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$	0.5
Here $\frac{\partial P}{\partial y} = e^x = \frac{\partial Q}{\partial x}$ so ω is closed	0.5
ω is closed on $\mathbb{R}^3$ so it is exact, therefore f exists.	0.5
Starting from $\frac{\partial f}{\partial x}(x,y) = e^x(y+5)$, we obtain $f(x,y) = e^x(y+5) + g(y)$	0.25 + 0.25
Hence $\frac{\partial f}{\partial y}(x,y) = e^x + \frac{dg}{dy}(y) = e^x + 3e^y$	0.25
This gives $g(y) = 3e^y + C$ and $f(x,y,z) = e^x(y+5) + 3e^y + C$	0.25 + 0.25

Exercise 5 - Uncertainties and variations (4 points)

1. $C = \frac{2\pi\varepsilon h}{ln(1+\frac{e}{R_1})} \approx \frac{S\varepsilon}{e}$	0.5
since $ln(1+\frac{e}{R_1}) \approx \frac{e}{R_1}$ and $S = 2\pi R_1 h$	0.5
2. $\delta C = \frac{\partial C}{\partial R_1}\delta R_1 + \frac{\partial C}{\partial R_2}\delta R_2 + \frac{\partial C}{\partial h}\delta h$	0.5
$\frac{\partial C}{\partial R_1} = \frac{2\pi\varepsilon h}{R_1 ln^2(\frac{R_2}{R_1})}$	0.5
$\frac{\partial C}{\partial R_2} = -\frac{2\pi\varepsilon h}{R_2 ln^2(\frac{R_2}{R_1})}$	0.5
$\frac{\partial C}{\partial h} = \frac{2\pi\varepsilon}{ln(\frac{R_2}{R_1})}$	0.25
Hence $\delta C = \frac{2\pi\varepsilon h}{R_1 ln^2(\frac{R_2}{R_1})}\delta R_1 - \frac{2\pi\varepsilon h}{R_2 ln^2(\frac{R_2}{R_1})}\delta R_2 + \frac{2\pi\varepsilon}{ln(\frac{R_2}{R_1})}\delta h$	0.25
3. $\Delta C = \lvert\frac{\partial C}{\partial R_1}\rvert\Delta R_1 + \lvert\frac{\partial C}{\partial R_2}\rvert\Delta R_2 + \lvert\frac{\partial C}{\partial h}\rvert\Delta h$	0.5
$\Delta C = \frac{2\pi\varepsilon h}{R_1 ln^2(\frac{R_2}{R_1})}\frac{a}{2} + \frac{2\pi\varepsilon h}{R_2 ln^2(\frac{R_2}{R_1})}\frac{a}{2} + \frac{2\pi\varepsilon}{ln(\frac{R_2}{R_1})}a$	0.5