INSTITUT NATIONAL DES SCIENCES APPLIQUÉES DE LYON

Département du Premier Cycle - SCAN1
MTES - Semester 2 **2016-2017**

TEST 1 14/04/17
DURATION 1H30

Warnings and advices
- **Documents, dictionaries, phones, and calculators are FORBIDDEN.**
- **The marking scheme is only given as an indication.**

EXERCISE 1 *(2 pts)*

1. Give the precise mathematical definition of an orientable surface $\mathcal{S}$.
2. Give an example of an orientable surface. (no justifications required!)
3. Give an example of an non-orientable surface and explain how one can build it.

EXERCISE 2 *(2 pts)*

Consider $\mathbb{R}^2$ with the orthonormal frame $(O, \overrightarrow{i}, \overrightarrow{j})$.
Consider a homogeneous triangular plate OAB, O being the origin of the frame and $A(0,1)$ and $B(1,0)$.
Compute the area of this plate **using a double integral in cartesian coordinates.**

EXERCISE 3 *(5 pts)*

We consider a half ball in the upper half space ($z \geqslant 0$) with volumetric density ρ given in spherical coordinates by $\rho = \rho_0(1 + \cos^2(\varphi))$.

1. Compute the mass of this half ball. integration by parts ?
2. Show that this half ball has 2 different vertical planes of symmetry that contain the origin O.
3. Determine the center of mass of this half ball.

EXERCISE 4 *(5 pts)*

We consider a container whose lateral surface is a surface of revolution given by the equation (in cylindrical coordinates) $z \in [0, 2a]$ and $r^2(z) - z^2 = 2a^2$ with $a \in \mathbb{R}^{+*}$.

1. ? Draw a picture of a slice of the container in a vertical plane containing the origin. Show the tangents of the curves at the boundaries. We recall that $\sqrt{2} \approx 1.4$ and $\sqrt{6} \approx 2.5$.
2. Compute the area of the lateral surface.
   You can use the following result with no justification : $\displaystyle\int_0^{2a} \sqrt{a^2 + z^2}\, \mathrm{d}z = \frac{\operatorname{argsh}(2) + 2\sqrt{5}}{2} a^2$
3. Compute the volume of the container. (We consider that the thickness of the wall is negligible!)

EXERCISE 5 *(6 pts)*

Nothalfafool, student at INSA LYON, has understood that the coordinates of the center of mass G can be obtained by simply averaging the values of the coordinates of all the points. However, he is wondering why the same formulas in polar coordinates have not been given to him :

$$G_r = \frac{1}{M} \iint_S r\,\sigma \; \mathrm{d}S \quad \text{and} \quad G_\theta = \frac{1}{M} \iint_S \theta\,\sigma \; \mathrm{d}S$$

where G_r and G_θ are the polar coordinates of the center of mass of a plate S with mass M, where σ is the area density and dS represents the infinitesimal area.

In this exercice, the value of the polar angle angle θ belongs to $[0, 2\pi[$.

His friend *Perceptive* is going to help him by testing his formulas on some disk shape plates.

1. Remind the formula of the cartesian coordinates G_x and G_y of the center of mass of a plate S with mass M, and area density σ using polar coordinates under the integral sign $\iint$.
2. What are the cartesian coordinates G_x and G_y of an homogeneous disk of radius $R > 0$ centered at the origin ? Deduce the polar radius of the center of mass.
3. Compute the G_r given by the formula of *Nothalfafool*. Conclude.
4. *Perceptive* proposes to *Nothalfafool* to test his formula on a portion of the disk for which $\theta \in [\frac{\pi}{3}, \frac{3\pi}{4}] \cup [\frac{5\pi}{4}, \frac{5\pi}{3}]$. Let's denote by S' this new plate.
   (a) Draw a picture of S'.
   (b) Determine the coordinates G_x and G_y for S'.
   (c) Compute the G_θ given by the formula of *Nothalfafool* for S' using a double integral in polar coordinates. Conclude !