INSTITUT NATIONAL DES SCIENCES APPLIQUÉES DE LYON

INSTITUT NATIONAL DES SCIENCES APPLIQUEES LYON

Département du Premier Cycle - SCAN1
MTES - Semester 1

2017-2018

TEST 1 17/10/17
DURATION 1H

Warnings and advices
- **Documents, dictionaries, phones, and calculators are FORBIDDEN.**
- **The marking scheme is only given as an indication.**

EXERCISE 1

$\Delta = (3+4i)^2 - 4(-1+5i) = -7+4+24i-20i = -3+4i = 1^2-2^2+2*1*2i = (1+2i)^2 \neq 0.$

Two solutions $z_1 = \dfrac{-3-4i+1+2i}{2} = -1-i$ and $z_1 = \dfrac{-3-4i-1-2i}{2} = -2-3i.$

EXERCISE 2

Let $\theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$, $n \in \mathbb{N}$ and $z = (1+ie^{i\theta})^n$.

1. $z = (1+ie^{i\theta})^n = (1+e^{i(\theta+\frac{\pi}{2})})^n = e^{in(\frac{\theta}{2}+\frac{\pi}{4})}\left(e^{-i\ n(\frac{\theta}{2}+\frac{\pi}{4})} + e^{in(\frac{\theta}{2}+\frac{\pi}{4})}\right)^n = e^{i\ n(\frac{\theta}{2}+\frac{\pi}{4})}\left(2\cos\left(\frac{\theta}{2}+\frac{\pi}{4}\right)\right)^n.$

   So $u = e^{i\ n(\frac{\theta}{2}+\frac{\pi}{4})}$.

2. If n is even, then $\left(2\cos\left(\frac{\theta}{2}+\frac{\pi}{4}\right)\right)^n \geqslant 0$ so $|z| = 2^n\cos^n\left(\frac{\theta}{2}+\frac{\pi}{4}\right)$ and $Arg(z) = n(\frac{\theta}{2}+\frac{\pi}{4})$

3. Since $\theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$, then $\frac{\theta}{2}+\frac{\pi}{4} \in [\frac{\pi}{2}, \pi]$ so $\cos\left(\frac{\theta}{2}+\frac{\pi}{4}\right) \leqslant 0$. Since n is odd, then $-\left(2\cos\left(\frac{\theta}{2}+\frac{\pi}{4}\right)\right)^n \geqslant 0$ and $z = -\left(2\cos\left(\frac{\theta}{2}+\frac{\pi}{4}\right)\right)^n e^{i\ n(\frac{\theta}{2}+\frac{\pi}{4})+i\pi}$

   So $|z| = -2^n\cos^n\left(\frac{\theta}{2}+\frac{\pi}{4}\right))^n$ and $Arg(z) = n(\frac{\theta}{2}+\frac{\pi}{4})+\pi$

EXERCISE 3

1. The barycenter G of (A, m_A) and (B, m_B) is a point $G \in \mathbb{R}^3$ s.t. $m_A\overrightarrow{GA} + m_B\overrightarrow{GB} = \overrightarrow{0}$.
2. $m_A + m_B \neq 0$.
3. $\overrightarrow{AC} = \dfrac{m_A}{m_A+m_B}\overrightarrow{AA} + \dfrac{m_B}{m_A+m_B}\overrightarrow{AB}$ so $C = G$.

EXERCISE 4

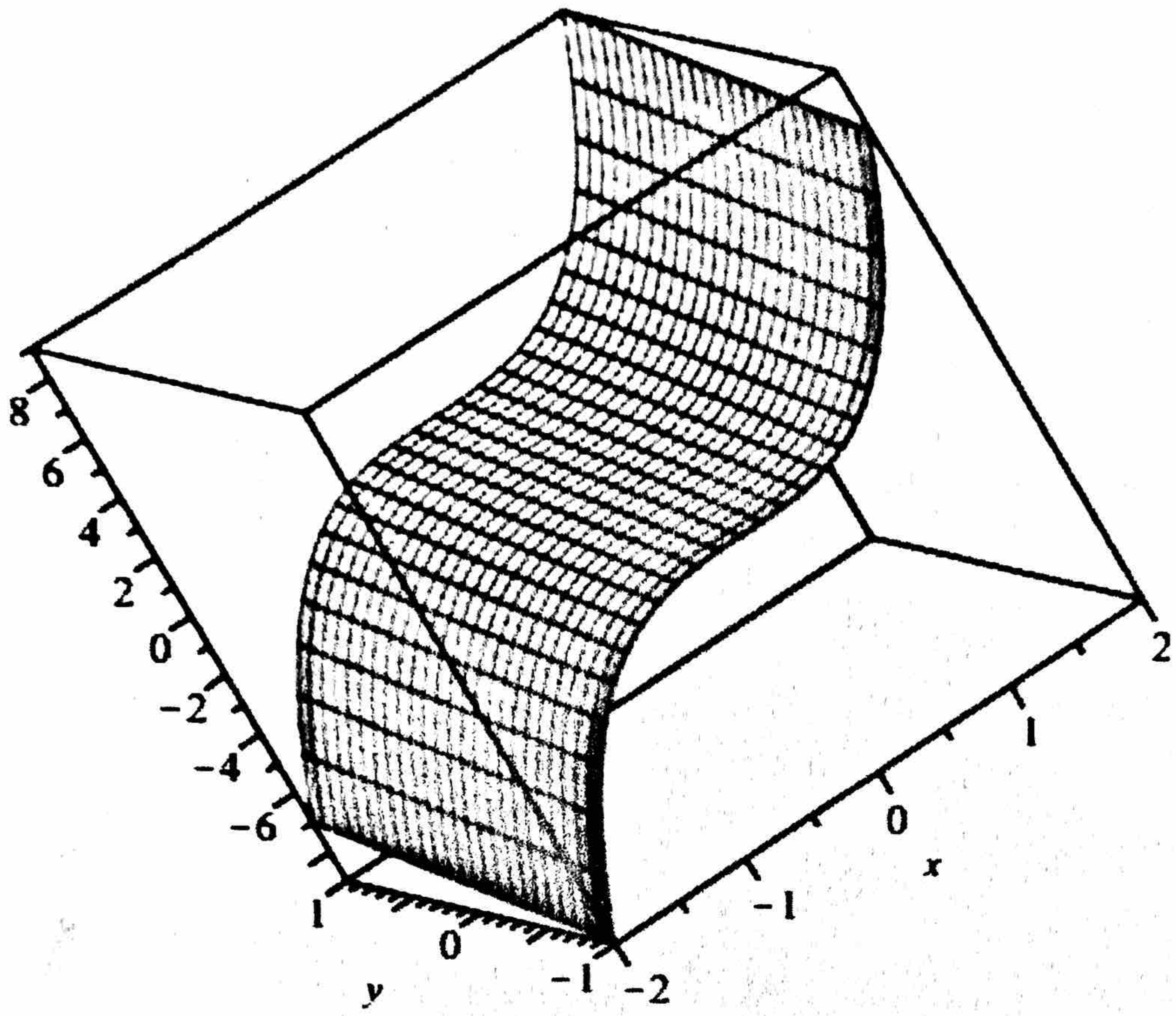
Consider the points $A(0,2,3)$ and $B(3,2,0)$ in $\mathbb{R}^3$.

1. $x_G = 0\frac{1}{3} + 3\frac{2}{3} = 2$, $x_G = 2\frac{1}{3} + 2\frac{2}{3} = 2$ and $x_G = 3\frac{1}{3} + 0\frac{2}{3} = 1$.

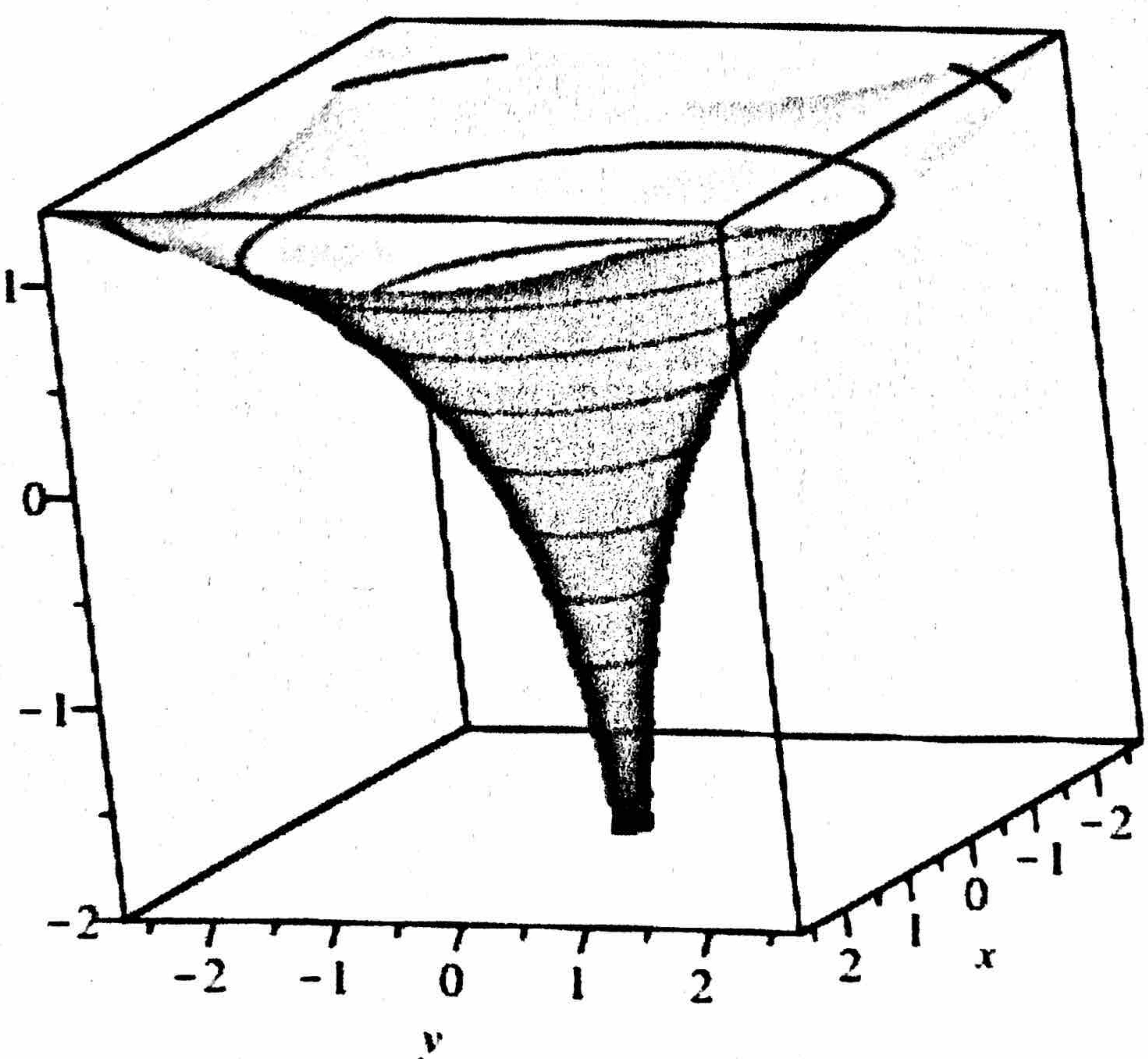
2. $V = |((\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OG}))| = |\begin{vmatrix} 0 & 3 & 2 \\ 2 & 2 & 2 \\ 3 & 0 & 1 \end{vmatrix}| = 0$ which was expected since the three vectors are coplanar. Indeed, $\overrightarrow{OG} = \frac{1}{3}\overrightarrow{OA} + \frac{2}{3}\overrightarrow{OB}$.

1. $P \in (AC)$ so $\overrightarrow{AP}$ and $\overrightarrow{AC}$ are collinear and since $\overrightarrow{AC} \neq 0$, there exists $u \in \mathbb{R}$ such that $\overrightarrow{AP} = u\overrightarrow{AC}$. The same for $\overrightarrow{DQ}$.
2. $\overrightarrow{PQ} \cdot \overrightarrow{AC} = (\overrightarrow{PA} + \overrightarrow{AD} + \overrightarrow{DQ}) \cdot \overrightarrow{AC}$
   And $\overrightarrow{PA} \cdot \overrightarrow{AC} = -u\overrightarrow{AC} \cdot \overrightarrow{AC} = -u\|\overrightarrow{AC}\|^2 = -2u$, $\overrightarrow{AD} \cdot \overrightarrow{AC} = (0,0,1) \cdot (1,0,1) = 1$ and $\overrightarrow{DQ} \cdot \overrightarrow{AC} = v\overrightarrow{DF} \cdot \overrightarrow{AC} = v(0,1,-1) \cdot (1,0,1) = -v$.
   Similarly $\overrightarrow{PQ} \cdot \overrightarrow{DF} = (\overrightarrow{PA} + \overrightarrow{AD} + \overrightarrow{DQ}) \cdot \overrightarrow{DF}$
   And $\overrightarrow{PA} \cdot \overrightarrow{DF} = -u\overrightarrow{AC} \cdot \overrightarrow{DF} = -u(1,0,1) \cdot (0,1,-1) = u$, $\overrightarrow{DC} \cdot \overrightarrow{DF} = (0,0,1) \cdot (0,1,-1) = -1$ and $\overrightarrow{DQ} \cdot \overrightarrow{DF} = v\overrightarrow{DF} \cdot \overrightarrow{DF} = v\|\overrightarrow{DF}\|^2 = 2v$.
3. Since (PG) is orthogonal to (AC) and (FD), we have that $\overrightarrow{PQ} \cdot \overrightarrow{AC} = \overrightarrow{PQ} \cdot \overrightarrow{DF} = 0$. So $2u + v = 1$ and $u + 2v = 1$ which lead to $u = v = \dfrac{1}{3}$ so $\overrightarrow{AP} = \dfrac{1}{3}\overrightarrow{AC} = \left(\dfrac{1}{3}, 0, \dfrac{1}{3}\right)$ and

   $\overrightarrow{AQ} = \overrightarrow{AD} + \overrightarrow{DQ} = (0,0,1) + \dfrac{1}{3}(0,1,-1) = \left(0, \dfrac{1}{3}, \dfrac{2}{3}\right)$
4. $\mathcal{A} = \dfrac{1}{2}\|\overrightarrow{AP} \wedge \overrightarrow{AQ}\| = \dfrac{1}{18}\|(-1,-2,1)\| = \dfrac{\sqrt{6}}{18}$.



Graph of function f



Graph of function g