INSTITUT NATIONAL DES SCIENCES APPLIQUÉES DE LYON INSA

Département du Premier Cycle - SCAN1
MTES - Semester 1 2017-2018

Test 1 17/10/17
Duration 1h

Warnings and advices
- **Documents, dictionaries, phones, and calculators are FORBIDDEN.**
- **The marking scheme is only given as an indication.**

Exercise 1 *(3 pts)*
Solve in $\mathbb{C}$ the equation $z^2 + (3+4i)z - 1 + 5i = 0$.

Exercise 2 *(6 pts)*
Let $\theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$, $n \in \mathbb{N}$ and $z = (1 + ie^{i\theta})^n$.

1. Find a complex number u such that $|u| = 1$ and $z = 2^n \cos^n\left(\frac{\theta}{2} + \frac{\pi}{4}\right) u$.
2. Determine the modulus and an argument of z if n is even.
3. Determine the modulus and an argument of z if n is odd.

Exercise 3 *(2 pts)*
Consider 2 weighted points (A, m_A) and (B, m_B) in $\mathbb{R}^3$ with $A \neq B$ and $m_A m_B \neq 0$.

1. Give the definition of a barycenter G of (A, m_A) and (B, m_B).
2. Give a necessary and sufficient condition for the existence of G.
3. Let $C \in \mathbb{R}^3$ such that $\overrightarrow{AC} = \dfrac{m_B}{m_A + m_B}\overrightarrow{AB}$. What can you say about the point C?

Exercise 4 *(2.5 pts)*
Consider the points $A(0,2,3)$ and $B(3,2,0)$ in $\mathbb{R}^3$.

3 2
2 2
0 1
3 2
2 2

1. Compute the coordinates of the barycenter G of $(A,1)$ and $(B,2)$.
2. Compute the volume of the parallelepiped generated by the vectors $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OG}$.

Exercise 5 *(6.5 pts)*
Consider in $\mathbb{R}^3$ the points $A(0,0,0)$, $D(0,0,1)$, $C(1,0,1)$ and $F(0,1,0)$.

We are looking for the coordinates of the points $P \in (AC)$ and $Q \in (FD)$ such that (PQ) is orthogonal to (AC) and (FD).

1. Justify that there exists $u, v \in \mathbb{R}$ such that $\overrightarrow{AP} = u\overrightarrow{AC}$ and $\overrightarrow{DQ} = v\overrightarrow{DF}$.
2. Using Chasles property show that $\overrightarrow{PQ} \cdot \overrightarrow{AC} = -2u + 1 - v$ and $\overrightarrow{PQ} \cdot \overrightarrow{DF} = u - 1 + 2v$.
3. Deduce from it that the coordinates of P and Q are respectively $\left(\frac{1}{3}, 0, \frac{1}{3}\right)$ and $\left(0, \frac{1}{3}, \frac{2}{3}\right)$.
4. Compute the area $\mathcal{A}$ of the triangle APQ.
