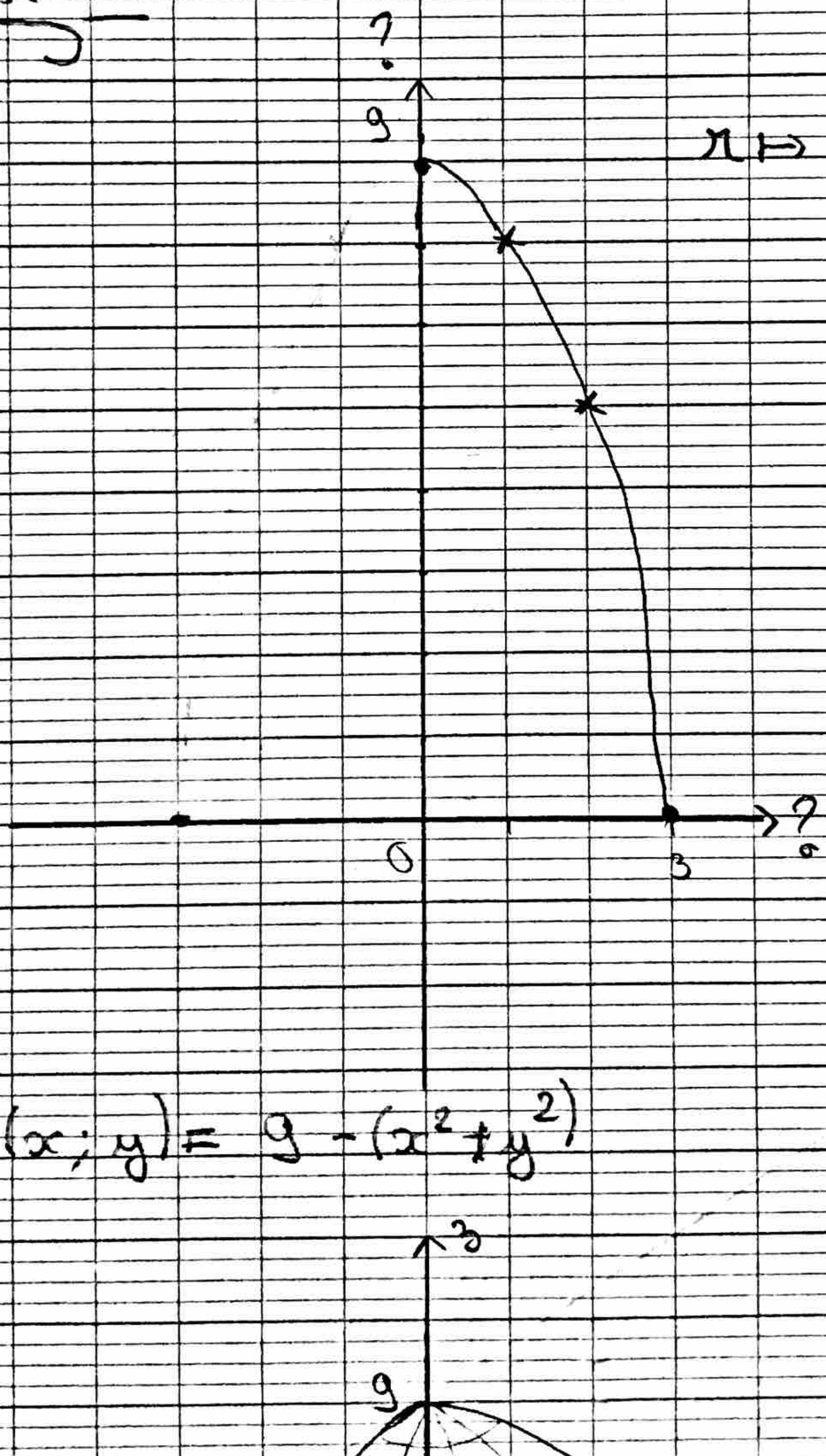
Naëlie
Giraud
61

JOMSI/MTES Test

$\frac{17,75}{23,5}$

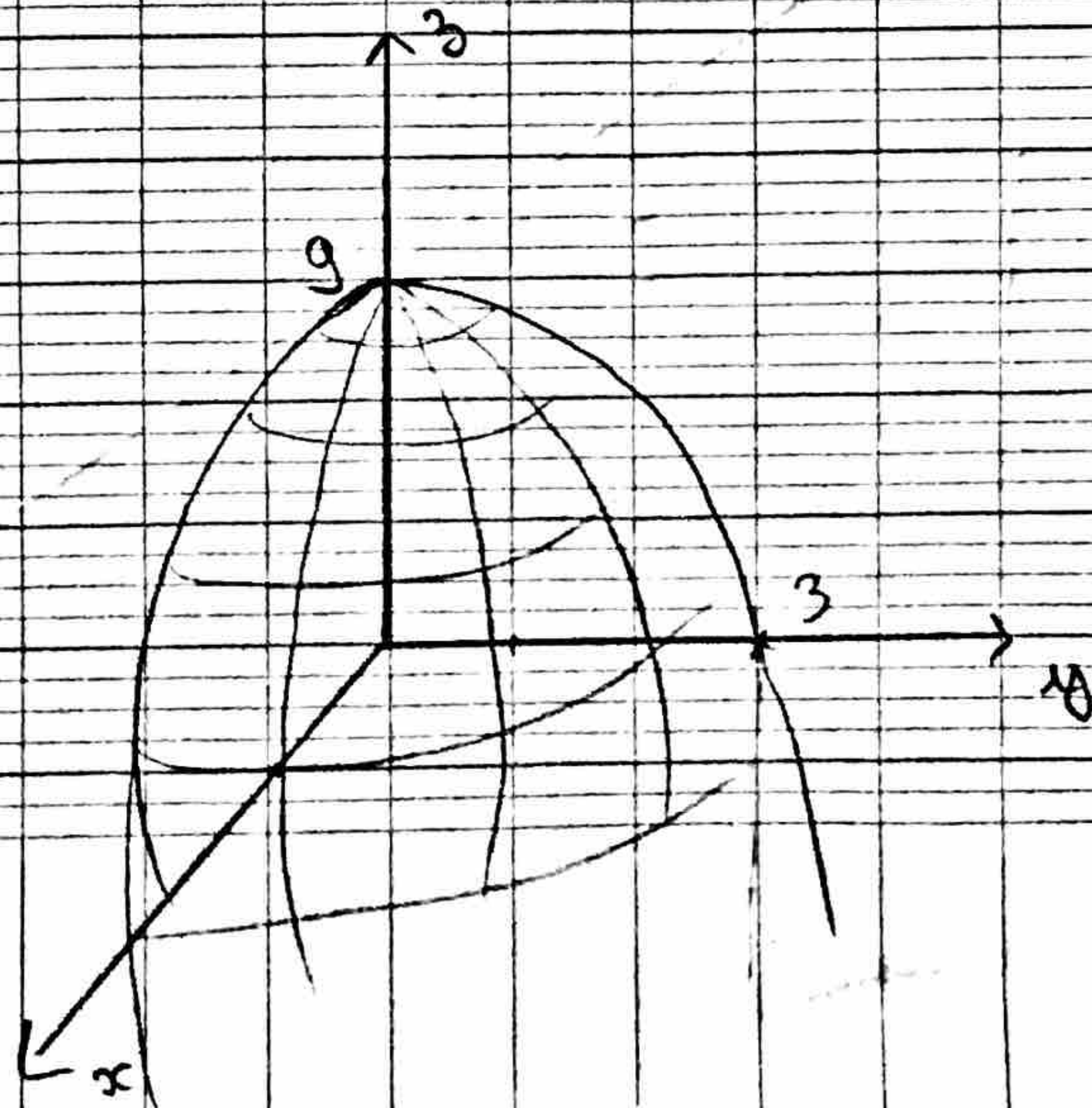
Exercise 1:

1) $x \mapsto 9 - x^2$ on $[0;3]$



0,5

2) $f(x;y) = 9 - (x^2 + y^2)$



0,5

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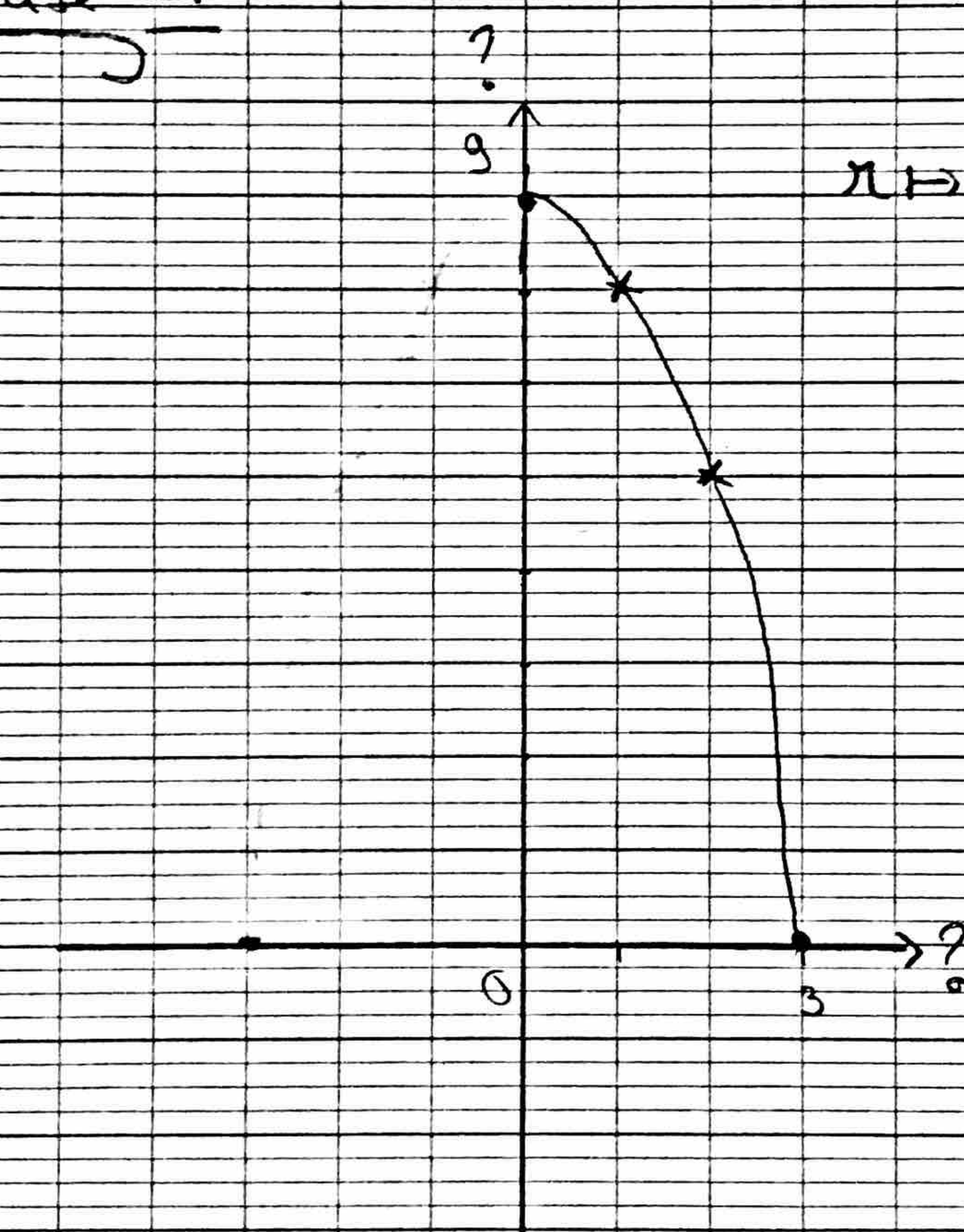

0,5

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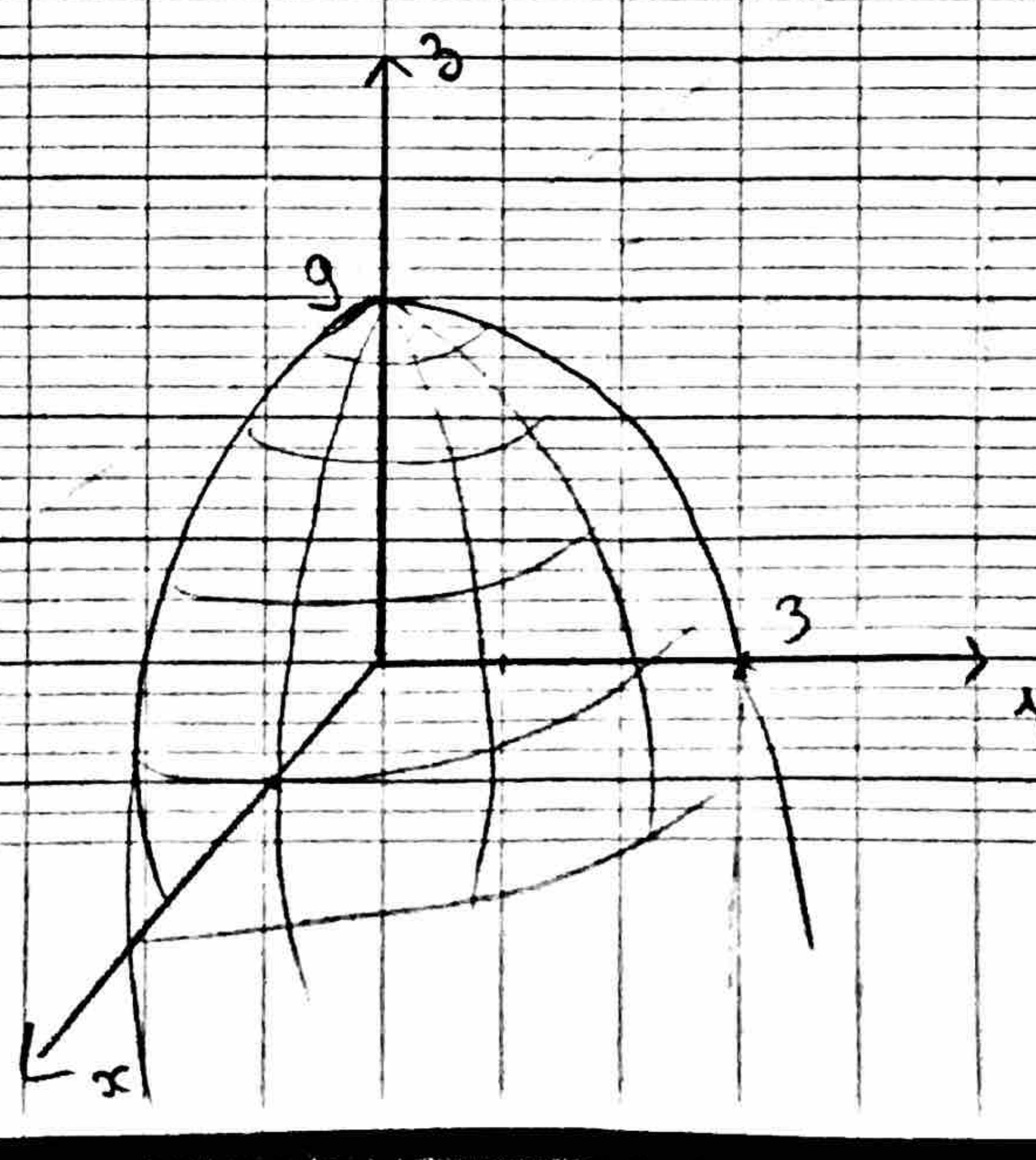

0,5

(c) $\vec{OM} = t\vec{e_r} + \cancel{1\vec{e_\theta}} + (3-t^2)\vec{e_z}$

(d) $\vec{v} = \frac{\partial \vec{OM}}{\partial t} = \vec{e_r} + \cancel{1\vec{e_\theta}} + -2t\vec{e_z}$

Exercise 2:

1) $\omega = dx - \frac{2y(x+z)}{1+y^2} dy + dz$

$$\frac{\partial\left(-\frac{2y(x+z)}{1+y^2}\right)}{\partial x} = \frac{-2y}{1+y^2} \quad \text{and} \quad \frac{\partial (1)}{\partial y} = 0$$

1 since $\frac{-2y}{1+y^2} \neq 0$ ω is not a closed form.

2) $\omega_1 = \underbrace{\varphi(y)\,dx}_{P} \underbrace{- \varphi(y)\frac{2y(x+3)}{1+y^2}}_{Q} dy + \underbrace{\varphi(y)\,dz}_{R}$

ω_1 is closed $\Leftrightarrow$ $\begin{cases} \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} & (1) \\ \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x} & (2) \\ \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y} & (3) \end{cases}$

Since φ only depends on φ let's take (1) not right, you should consider all 3 equations

0,5 $\frac{\partial P}{\partial y} = \varphi'(y)$ and $\frac{\partial Q}{\partial x} = \frac{-\varphi(y)\,2y}{1+y^2}$

Hence ω_1 is closed $\Leftrightarrow$ $\varphi'(y) = -\varphi(y) \cdot \frac{2y}{1+y^2}$.

3) $\omega_1 = \frac{1}{1+y^2}\,dx - \frac{2y\,(x+z)}{(1+y^2)^2}\,dy + \frac{1}{1+y^2}\,dz$

$$= \varphi_1(y)\,dx - \varphi_1(y) \times \frac{2y\,(x+z)}{1+y^2} + \varphi_1(y)\,dz$$

with $\varphi_1 = \frac{1}{1+y^2}$

0,75 but $\varphi_1' = -\frac{2y}{(1+y^2)^2} = \frac{-2y}{(1+y^2} \times \varphi_1$

Hence φ_1 responds to the criteria establish in question 2 and so ω_1 is a closed form.

4) Let's find f such that $df = \omega_1$

$$df = \frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy + \frac{\partial f}{\partial z}\,dz$$

Hence $\frac{\partial f}{\partial x} = \frac{1}{1+y^2} \xrightarrow[w/x]{\text{integrate}} \frac{x}{1+y^2} + C_1$

with C_1 a constant with respect to x

derivate what we obtain w/ z $\longrightarrow$ $0 + \frac{\partial C_1}{\partial z}$ but since $\frac{\partial f}{\partial z} = \frac{1}{1+y^2}$

we have $C_1 = \frac{z}{1+y^2} + C_2$

with C_2 a constant with respect to z

Hence $f = \frac{x}{1+y^2} + \frac{z}{1+y^2} + C_2$

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derivate f w/ y $\longrightarrow -x \times \frac{2y}{(1+y^2)^2} - 3 \times \frac{2y}{(1+y^2)^2} + \frac{\partial C_2}{\partial y}$

$$= \frac{-2y\,(x+3)}{(1+y^2)^2} + \frac{\partial C_2}{\partial y} = \frac{\partial f}{\partial y}$$

1,5

Hence C_2 is a constant.

and $\boxed{f = \frac{x}{1+y^2} + \frac{3}{1+y^2} + C_2}$

Exercise 3:

1,5

$$\begin{cases} x = r \sin\theta \cos\varphi \\ y = r \sin\theta \sin\varphi \\ z = r\cos\theta \end{cases}$$

Exercise 4:

1) $\ell = r \cos(\theta) \sin(\varphi)$

hence $\boxed{\delta\ell = \cos\theta \sin\varphi \cdot \delta r + -r\sin\theta \sin\varphi \cdot \delta\theta + r\cos(\theta)\cos(\varphi) \cdot \delta\varphi}$

2

2) when $r = 1$ $\theta = \frac{\pi}{4}$ and $\varphi = \frac{\pi}{2}$

then $\delta\ell = \left(\frac{\sqrt{2}}{2} \times 1\right) \times 10^{-2} - \left(1 \times 1 \times \frac{\sqrt{2}}{2}\right) \pi \cdot 10^{-2} + 0$

0,5

$$= \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}\pi}{2}\right) \cdot 10^{-2} = (1-\pi)\frac{\sqrt{2}}{2} \cdot 10^{-2}$$

3) a) $\boxed{\Delta l = |\cos\theta \sin\varphi| \cdot \Delta r + |-r\sin\theta \sin\varphi| \cdot \Delta\theta + |r\cos\theta\cos\varphi| \cdot \Delta\varphi}$

1

b) when $r = 1$, $\theta = \frac{\pi}{2}$ $\varphi = -\frac{\pi}{4}$

$\Delta r = 10^{-2}$ $\Delta\theta = \Delta\varphi = 10^{-2}$

$\Delta l = 0 \times \Delta r + |-1 \times 1 \times -\frac{\sqrt{2}}{2}| \cdot 10^{-2} + |1 \times 0 \times \frac{\sqrt{2}}{2}| \cdot 10$

0,5 $\boxed{\Delta l = \frac{\sqrt{2}}{2} \cdot 10^{-2}}$

Exercise 5:

1) a) $\frac{\partial f}{\partial y}(-2;0) \cancel{\leq} 0$ (since $f \nearrow$ in y direction)

0,25 $\frac{\partial f}{\partial x}(0;1) = 0$

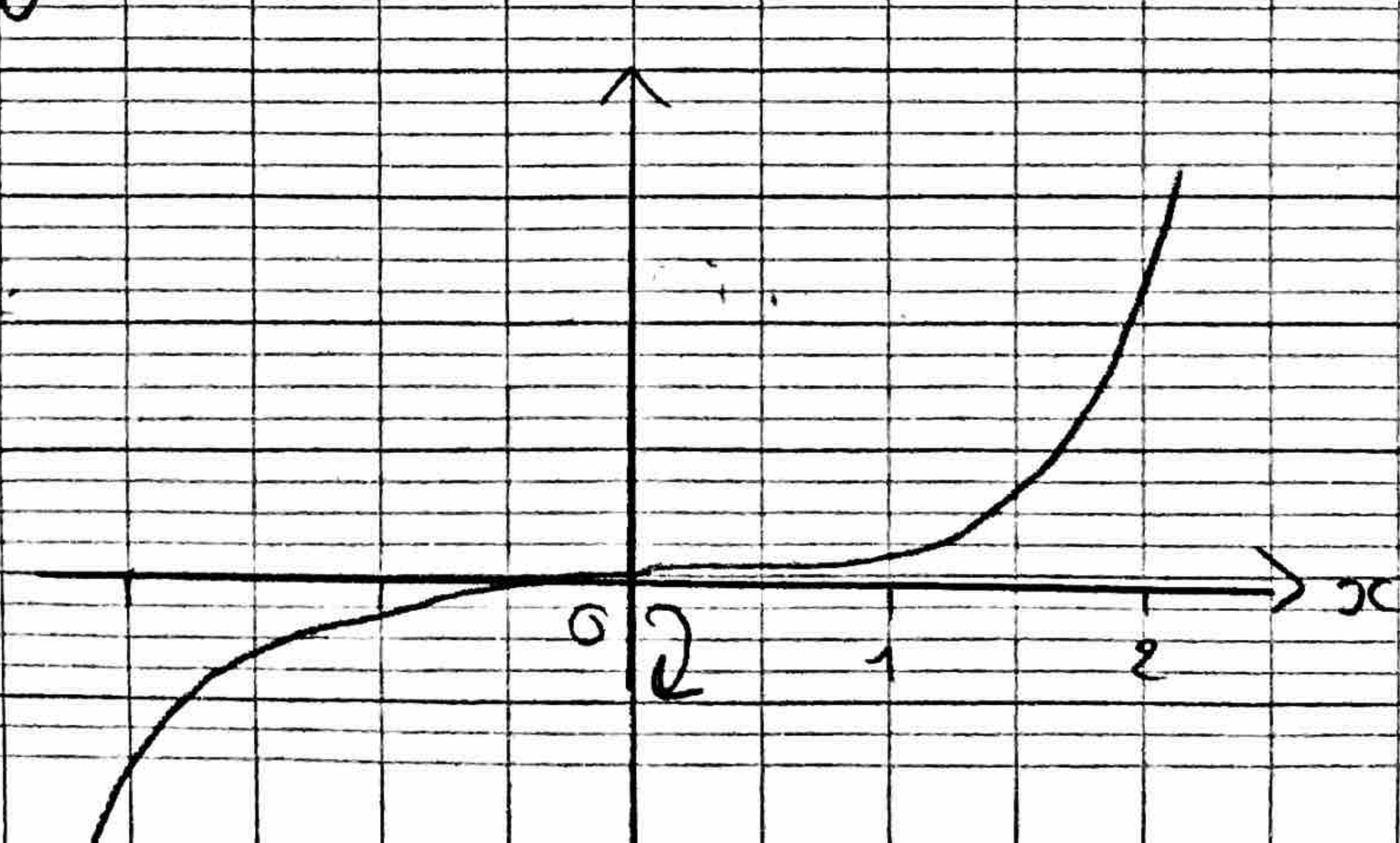
b) $f(0;0) \simeq 0$ $f(-2;-1) \simeq -9$ $f(-2;1) \simeq -7$

1,25 $f(2;1) \simeq \cancel{9}$

$f(x;y) = x^3 + y$.

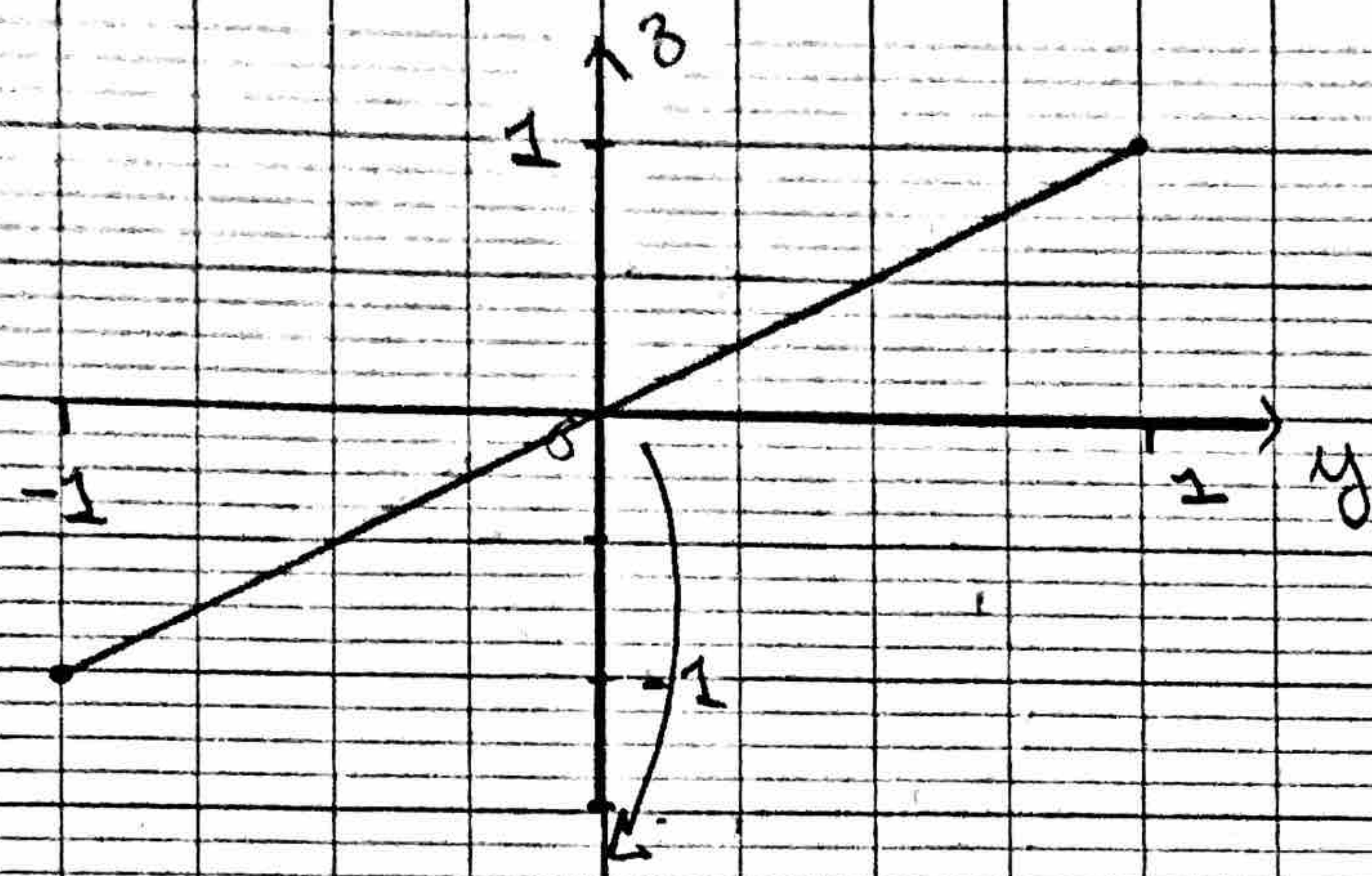
c)

0,25



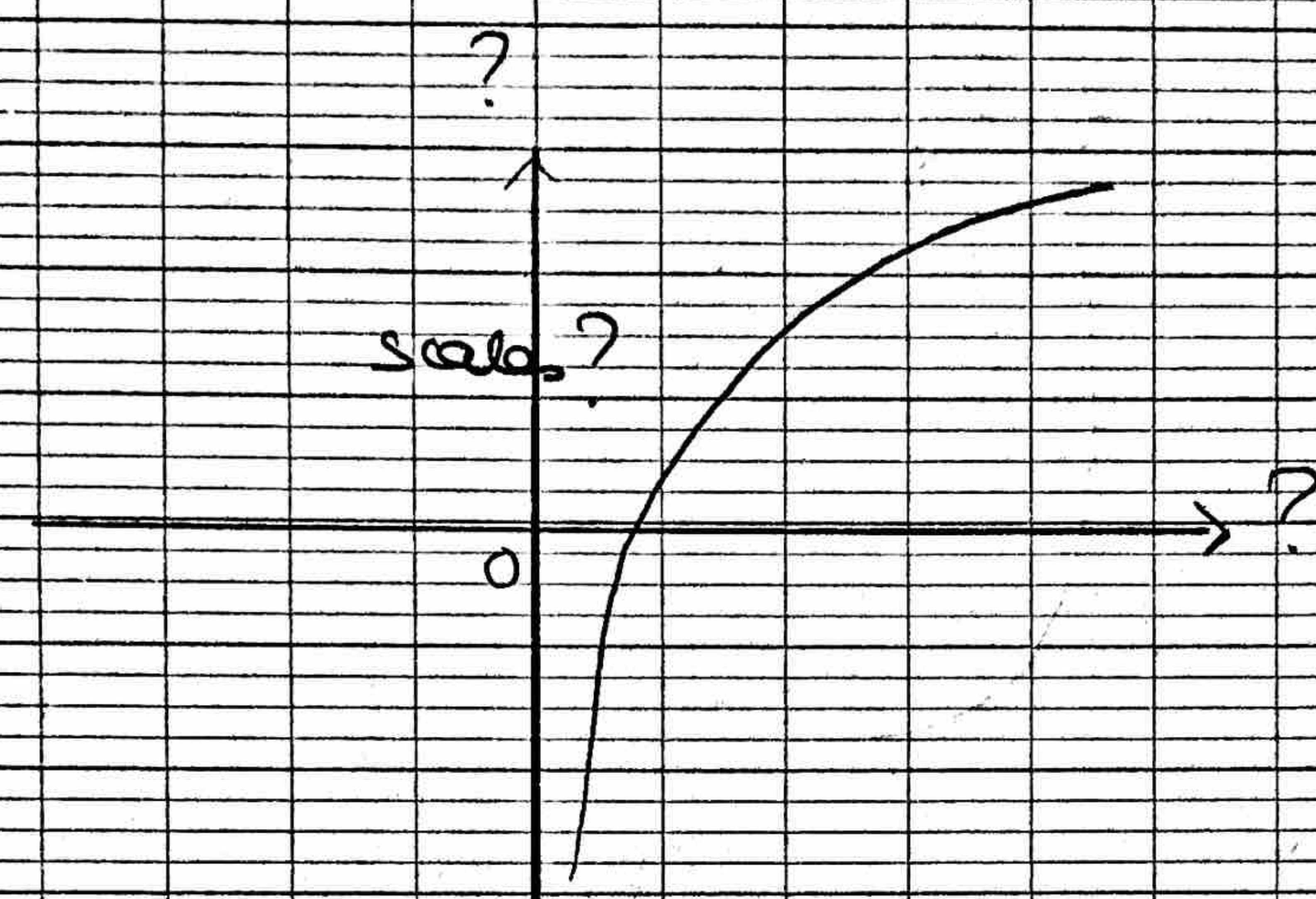
d)

0,25



2)a)

0,25



0,25 (b) $g(e;0) \simeq 1$

$g(e;e) \simeq \cancel{1}$

(c) $\ln(2) = \ln(e+2-e)$ Let $f(x) = \ln x$

$f((e)+(2-e)) \simeq f(e) + f'(e) \times (2-e)$

$= \ln(e) + \dfrac{2-e}{e}$

1,5

$= \dfrac{1 \times e + 2 - e}{e} = \dfrac{2}{e} \approx 0,740$

$\ln(\sqrt{2}\, e) = \ln(\sqrt{2}) + \ln(e) = \frac{1}{2}\ln 2 + 1 = 0,370 + 1$

$= 1,370$

0,75

(d) $g(x;y) = \ln\left(\sqrt{x^2+y^2}\right)$ (Use of squares for axis of revolution)

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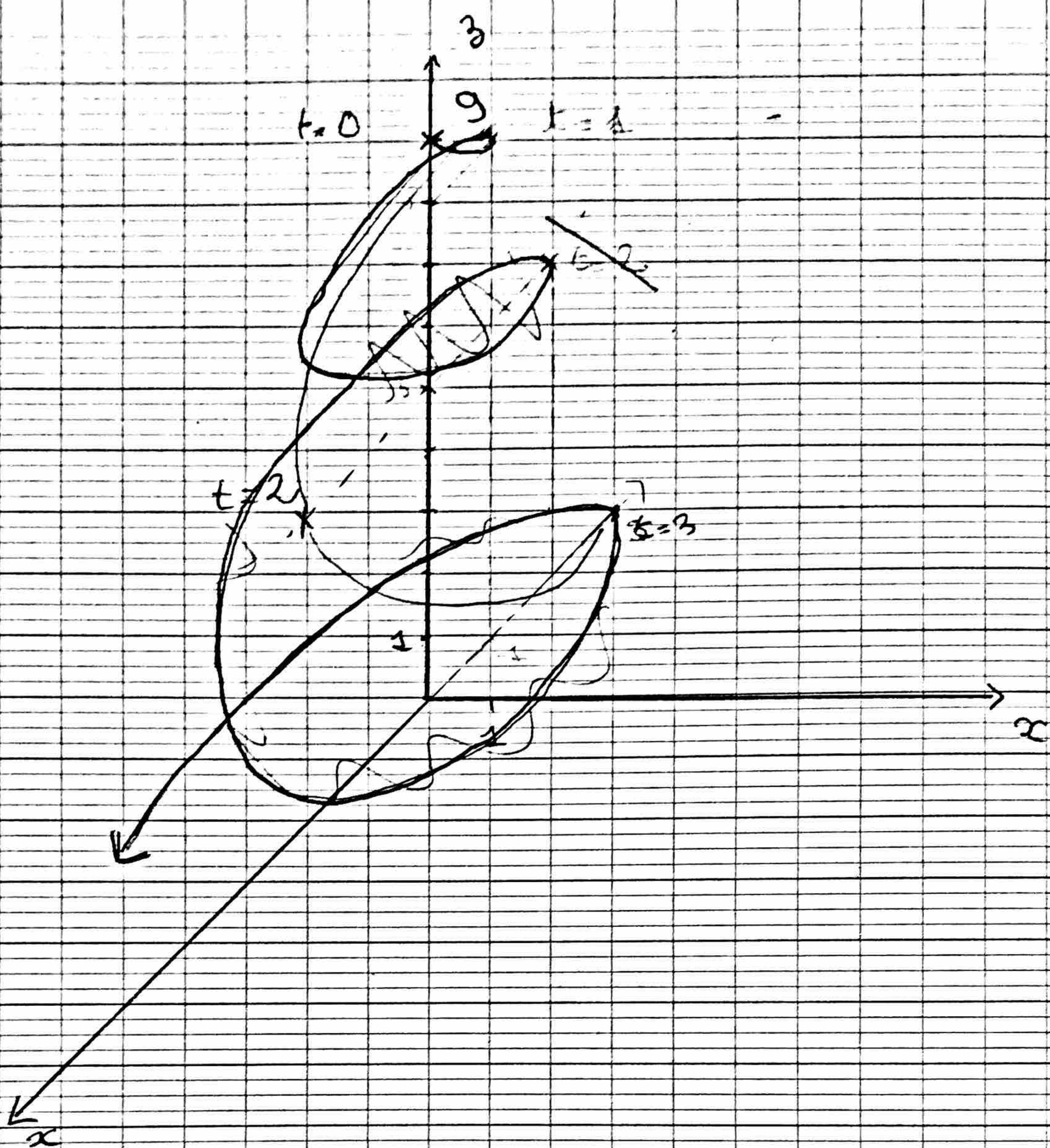
Question 4) b) Exercise 1:



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at $t=0$: $x=0$ $y=0$ $z=9$

at $t=1$ $x=1\times\cos\pi=-1$

$y=1\times\sin\pi=0$

$z=9-1=8$

at $t=2$ $x=2\cos(2\pi)=2$

$y=2\sin(2\pi)=0$

$z=9-4=5$

at $t=3$ $x=3\cos(3\pi)=-3$

$y=3\sin(3\pi)=0$

$z=0$