INSTITUT NATIONAL DES SCIENCES APPLIQUÉES DE LYON

Département du Premier Cycle - SCAN - First

IE 2 MTES - Duration 1h30

Warnings and advices
- **Documents, dictionaries, phones, and calculators are FORBIDDEN.**
- **The marking scheme is only given as an indication.**

EXERCISE 1 *(5 pts)*

In this exercice, **No justifications are required !** except at questions 4)a) and 4)d).

1. Sketch the graph of the function $r \mapsto 9 - r^2$ on $[0, 3]$.
2. Sketch the graph of the function : $f(x, y) = 9 - (x^2 + y^2)$ on an appropriate domain.
3. Let $(O, \vec{i}, \vec{j}, \vec{k})$ be an orthonormal frame of $\mathbb{R}^3$. Let r, θ and z be the cylindrical coordinates.
   (a) Recall the expression of the cartesian coordinates x, y, z with respect to the cylindrical coordinates r, θ, z.
   (b) Recall the expression of the vectors $\vec{e_r}$, $\vec{e_\theta}$ and $\vec{e_z}$ of the local cylindrical frame with respect to $\vec{i}$, $\vec{j}$ and $\vec{k}$.
4. Consider the following parametric curve in cylindrical coordinates :

$$\left\{\begin{array}{lcl} r(t) & = & t \\ \theta(t) & = & \pi t \\ z(t) & = & 9 - t^2 \end{array}\right. \qquad t \in [0, 3]$$

   It represents the trajectory of a point $M(t)$.
   (a) Show that this parametric curve is on the graph of f.
   (b) Give a representation of the trajectory of $M(t)$. Place the points for $t = 0$, $t = 1$, $t = 2$ and $t = 3$.
   (c) Express the vector $\overrightarrow{OM}(t)$ in the cylindrical frame.
   (d) Compute the velocity $\vec{v}(t) = \overrightarrow{OM}'(t)$ in the cylindrical frame.

EXERCISE 2 *(5 pts)*

1. Show that the differential form $\omega = \,\mathrm{d}x - \dfrac{2y(x+z)}{1+y^2}\,\mathrm{d}y + \,\mathrm{d}z$ on $\mathbb{R}^3$ is not a closed form.
2. Consider now the differential form $\omega_1 = \varphi(y)\omega$:

$$\omega_1 = \varphi(y)\,\mathrm{d}x - \varphi(y)\frac{2y(x+z)}{1+y^2}\,\mathrm{d}y + \varphi(y)\,\mathrm{d}z$$

   where φ is a function defined on $\mathbb{R}$.
   Show that ω_1 is a closed form on $\mathbb{R}^3$ iff $\forall y \in \mathbb{R}, \varphi'(y) = -\dfrac{2y}{1+y^2}\varphi(y)$.
3. Deduce from the previous question that $\omega_1 = \dfrac{1}{1+y^2}\,\mathrm{d}x - \dfrac{2y(x+z)}{(1+y^2)^2}\,\mathrm{d}y + \dfrac{1}{1+y^2}\,\mathrm{d}z$ is an exact form on $\mathbb{R}^3$.
4. Find a function f defined on $\mathbb{R}^3$ such that $df = \omega_1$.

Exercise 3 *(1.5 pts)*

Give the expression of the cartesian coordinates x, y, z with respect to the spherical coordinates r, θ, φ. (No justification required)

Exercise 4 *(3.5 pts)*

Consider a quantity l express in spherical coordinates by : $l = r\cos(\theta)\sin(\varphi)$. **So l can be seen as a function of the three variables r, θ, φ.**

1. Express the variation δl of l with respect to the variations δr, $\delta\theta$ and $\delta\varphi$ of r, θ, φ.
2. Compute δl at the point of spherical coordinates $(1, \frac{\pi}{4}, \frac{\pi}{2})$ when $\delta r = 10^{-2}$, $\delta\theta = \pi 10^{-2}$ and $\delta\varphi = \pi 10^{-2}$.
3. Now a point of the space is given by its spherical coordinates with uncertainties $\Delta r, \Delta\theta, \Delta\varphi$.
   (a) Express the uncertainty Δl.
   (b) Compute Δl when $r = 1, \theta = \frac{\pi}{2}, \varphi = -\frac{\pi}{4}$ and $\Delta r = 10^{-2}$ and $\Delta\theta = \Delta\varphi = 10^{-2}$

Exercise 5 *(5 pts)*

On the next page are the graphs of two functions f and g.

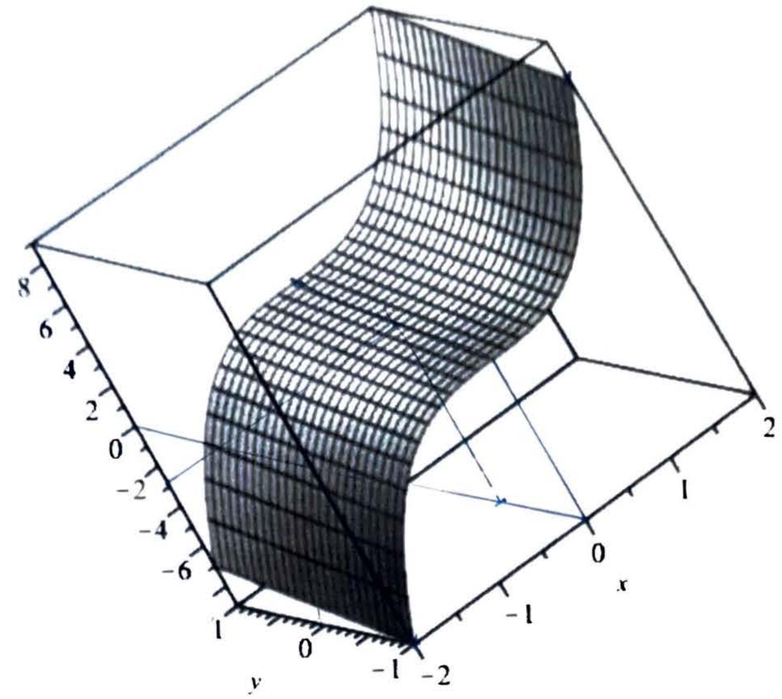
The function f is defined on $\mathbb{R}^2$ and the function g is defined on $\mathbb{R}^2 \setminus \{(0,0)\}$.

The graph of f is given for $(x, y) \in [-2, 2] \times [-1, 1]$.

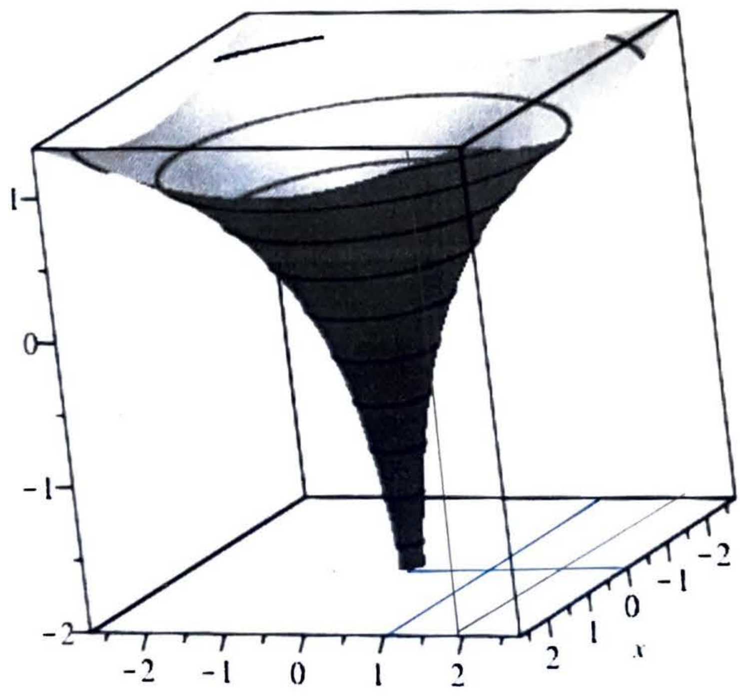
The graph of g is given for $(x, y) \in [-e, e] \times [-e, e]$ and $\lim\limits_{(0,0)} g(x, y) = -\infty$

In this exercice, you don't need to justify your answer except when it is specified to do so!

1. (a) Suppose that f is differentiable at $(-2, 0)$, what is the sign of $\frac{\partial f}{\partial y}(-2, 0)$? What is the value of $\frac{\partial f}{\partial x}(0, -1)$?
   (b) Give an approximate value of f at $(0, 0)$ $(-2, -1)$, $(-2, 1)$ and $(2, -1)$. Propose a simple expression for the function f that matches these values.
   (c) Draw the graph of the partial map of f with respect to x at the point $(0, -1)$.
   (d) Draw the graph of the partial map of f with respect to y at the point $(-2, 0)$.
2. (a) Draw the graph of the partial map of g at the point $(0, 1)$ with respect to y $(y > 0)$.
   (b) Give an approximate value for $g(e, 0)$ and $g(e, e)$.
   (c) Considering that $\ln(2) = \ln(e + 2 - e)$, show that a first order approximation of $\ln(2)$ is $\frac{2}{e}$. Maple gives $\frac{2}{2.7} \approx 0.740$. Deduce an approximation value of $\ln(\sqrt{2}e)$.
   (d) Propose a simple expression for the function g coherent with your previous answers.



Graph of function f



Graph of function g