

Warnings and advices

- Documents, dictionaries, phones, and calculators are FORBIDDEN.
- The marking scheme is only given as an indication.
- In this test, you can use directly the usual formulas : area of a disk, of a sphere, volume of a ball, etc... if you know them by heart.

EXERCISE 1 (6 pts)

1. We consider 2 circles :

- C_1 of center $(1, 1)$ and radius 1.
- C_2 of center $(0, 1)$ and radius $\sqrt{2}$.

- (a) Draw, on the same picture, those two circles.
- (b) Give, for each circle, its equation.

2. We consider the domain D defined by the 3 following conditions :

- D is inside the circle C_2 .
- D is in the half plane $x \geq 0$.
- D is under the circle C_1 .

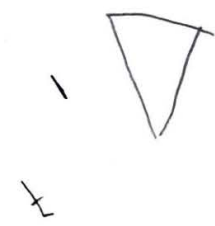
- (a) Represent the domain D on your previous drawing.
- (b) Is D a domain normal with respect to x ? Is D a domain normal with respect to y ? (No Justification required!)
- (c) Explain why $D = \{(x, y) | 0 \leq x \leq 1, 1 - \sqrt{2 - x^2} \leq y \leq 1 - \sqrt{1 - (x - 1)^2}\}$ (you can justify some part of your answer from your drawing).

3. (a) Show that $\int_0^1 \sqrt{2 - x^2} dx = \frac{1}{2} + \frac{\pi}{4}$.

(b) We assume that $\int_0^1 \sqrt{1 - (x - 1)^2} dx = \frac{\pi}{4}$. Compute the area of D .

(c) BONUS : Show that $\int_0^1 \sqrt{1 - (x - 1)^2} dx = \frac{\pi}{4}$.

$\int \sqrt{2 - x^2} dx = \frac{1}{2} \int \sqrt{2 - u^2} du$



Since $\sin \theta = \frac{y}{r}$

EXERCISE 2 (5 pts)

Consider a disk D in \mathbb{R}^2 of radius R centered at the origin with area density $\sigma = \sigma_0 \theta (2\pi - \theta)$ where θ is the polar angle such that $\theta \in [0, 2\pi]$ and σ_0 a constant.

1. Compute the mass of this disk.
2. Compute the coordinates of the center of mass. (Hint : Dont be afraid of IbPs!)

$$e^z = x \implies \frac{dz}{dx} = e^z \implies dz = e^z dx$$

EXERCISE 3 (6 pts)

1. Consider the curve $z = \ln(x)$ for $x \in [1, e]$. Draw this curve in the plane Oxz .
2. We rotate this curve around the Oz axis and obtain a surface of revolution S .
 - (a) Sketch this surface.
 - (b) We denote $I = \int_1^e \sqrt{1+u^2} du$. Express the area of S with respect to I . (Don't try to compute I ... the result is not pretty...)
3. We suppose that this surface S has an area density $\sigma = \frac{1}{\sqrt{1+e^{2z}}}$.
 - (a) Compute the mass of the surface S .
 - (b) Compute the coordinates of its center of mass.
4. We consider a box B whose lateral surface is S and which is closed by 2 horizontal planes of equation $z = 0$ and $z = 1$. Compute the volume of the box.

EXERCISE 4 (3 pts)

Compute the coordinates of the center of mass of a homogeneous half upper ball of radius R using spherical coordinates.