INSTITUT NATIONAL DES SCIENCES APPLIQUÉES DE LYON

Département du Premier Cycle - SCAN - First

IE 1- 26/04/18 MTES - Duration 1h30

Warnings and advices
- **Documents, dictionaries, phones, and calculators are FORBIDDEN.**
- **The marking scheme is only given as an indication.**
- **In this test, you can use directly the usual formulas : area of a disk, of a sphere, volume of a ball, etc... if you know them by heart.**

EXERCISE 1 *(6 pts)*

1. We consider 2 circles :
   - $\mathcal{C}_1$ of center $(1,1)$ and radius 1.
   - $\mathcal{C}_2$ of center $(0,1)$ and radius $\sqrt{2}$.

   (a) Draw, on the same picture, those two circles.

   (b) Give, for each circle, its equation.

$\sqrt{2-\tan^2\theta}\quad \frac{1}{\cos^2\theta}\, d\theta$

2. We consider the domain D defined by the 3 following conditions :
   - D is inside the circle $\mathcal{C}_2$.
   - D is in the half plane $x \geqslant 0$.
   - D is under the circle $\mathcal{C}_1$.

   (a) Represent the domain D on your previous drawing.

   (b) Is D a domain normal with respect to x ? Is D a domain normal with respect to y ? (No Justification required !)

   (c) Explain why $D = \{(x,y) | 0 \leqslant x \leqslant 1, 1-\sqrt{2-x^2} \leqslant y \leqslant 1-\sqrt{1-(x-1)^2}\}$ (you can justify some part of your answer from your drawing).

$\sin\theta = x$

3. (a) Show that $\displaystyle\int_0^1 \sqrt{2-x^2}\, \mathrm{d}x = \frac{1}{2} + \frac{\pi}{4}$.

   (b) We assume that $\displaystyle\int_0^1 \sqrt{1-(x-1)^2}\, \mathrm{d}x = \frac{\pi}{4}$. Compute the area of D.

   (c) BONUS : Show that $\displaystyle\int_0^1 \sqrt{1-(x-1)^2}\, \mathrm{d}x = \frac{\pi}{4}$.

EXERCISE 2 *(5 pts)*

Consider a disk D in $\mathbb{R}^2$ of radius R centered at the origin with area density $\sigma = \sigma_0\theta(2\pi - \theta)$ where θ is the polar angle such that $\theta \in [0, 2\pi]$ and σ_0 a constant.

1. Compute the mass of this disk.
2. Compute the coordinates of the center of mass. (*Hint* : Dont be afraid of IbPs !)

$e^z = x$

$\frac{dx}{dz} = e^z \qquad e^z \quad dz$

EXERCISE 3 *(6 pts)*

1. Consider the curve $z = \ln(x)$ for $x \in [1, e]$. Draw this curve in the plane Oxz.
2. We rotate this curve around the Oz axis and obtain a surface of revolution S.
   (a) Sketch this surface.
   (b) We denote $I = \displaystyle\int_1^e \sqrt{1+u^2}du$. Express the area of S with respect to I. (Don't try to compute I... the result is not pretty...)
3. We suppose that this surface S has an area density $\sigma = \dfrac{1}{\sqrt{1+e^{2z}}}$.
   (a) Compute the mass of the surface S.
   (b) Compute the coordinates of its center of mass.
4. We consider a box B whose lateral surface is S and which is closed by 2 horizontal planes of equation $z = 0$ and $z = 1$. Compute the volume of the box.

EXERCISE 4 *(3 pts)*

Compute the coordinates of the center of mass of a homogeneous half upper ball of radius R using spherical coordinates.
