$$= \frac{\lambda^5 + 2\lambda^3 + \lambda - \lambda^2 - \lambda^5 - 2\lambda^3 - \lambda + \lambda^2}{(\lambda^2+1)^2}$$

$$= \frac{0}{(\lambda^2+1)^2} = 0$$

Thus we have $\overrightarrow{HC} . \overrightarrow{HQ} = 0$.
We deduce that the straight lines (HC) and (HQ) are perpendicular (they cross in H)

4 Exercise 4) $x^2 + (-1+i)x + 2 - 2i \quad x \in \mathbb{C}$

$$\Delta = (-1+i)^2 - 4 \times 1 \times (2-2i)$$
$$= 1 - 2i - 1 - 8 + 8i = -8 + 6i$$

Let $\delta \in \mathbb{C}$ st $\delta^2 = \Delta$ with $\delta = a + ib \quad a, b \in \mathbb{R}$

$$\begin{cases} a^2 - b^2 = -8 \\ 2ab = 6 \\ a^2 + b^2 = \sqrt{(-8)^2 + 6^2} \end{cases} \Leftrightarrow \begin{cases} a^2 = b^2 - 8 \\ a \text{ and } b \text{ of same sign} \\ 2b^2 - 8 = 10 \end{cases}$$

$$\Leftrightarrow \begin{cases} a = 1 \text{ or } a = -1 \\ a \text{ and } b \text{ of same sign} \\ b = 3 \text{ or } b = -3 \end{cases} \Leftrightarrow \begin{cases} \delta = 1 + 3i \\ \text{or } \delta = -1 - 3i \end{cases}$$

Thus we have:

$$x_1 = \frac{1 - i + 1 + 3i}{2} = 1 + i$$

$$x_2 = \frac{1 - i - 1 - 3i}{2} = -2i$$

$$S = \{ 1+i \, ; \, 2i \}$$

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MTES (IE n°1)

October 24th

18,50

Exercise 1) 2 $O(0,0,0) \quad \overrightarrow{OA}(1,0,1) \quad \overrightarrow{OB}(1,2,0) \quad \overrightarrow{OC}(-1,2,4)$

$$V_{\text{parallelepiped}} = |((\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}))| = |\overrightarrow{OA} . (\overrightarrow{OB} \wedge \overrightarrow{OC})|$$

With $\overrightarrow{OB} \wedge \overrightarrow{OC} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \times 4 - 2 \times 0 \\ 0 \times -1 - 4 \times 1 \\ 1 \times 2 - (-1) \times 2 \end{pmatrix} = \begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix}$

Thus $V_{\text{parallelepiped}} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix} = 1 \times 8 - 4 \times 0 + 1 \times 4 = 1$

Exercise 2) 1 $(A_1, \underbrace{1}_{m_1}) \quad (A_2, \underbrace{-2}_{m_2})$

$$\overrightarrow{OG} = \frac{m_1}{m_1 + m_2} \overrightarrow{OA_1} + \frac{m_2}{m_1 + m_2} \overrightarrow{OA_2}$$

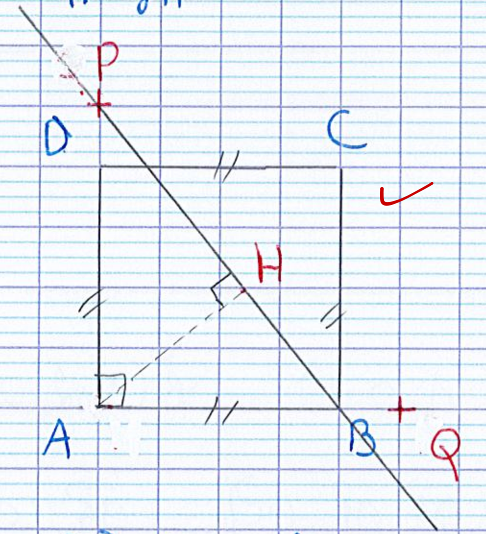
$$\Leftrightarrow \overrightarrow{OG} = -\overrightarrow{OA_1} + 2\overrightarrow{OA_2}$$

(The coordinates of a point M equal the coordinates of $\overrightarrow{OM}$ (with O the origin))

$$\boxed{\Leftrightarrow \begin{cases} x_G = -x_{A_1} + 2x_{A_2} \\ y_G = -y_{A_1} + 2y_{A_2} \end{cases}}$$

Exercise 3) $A(0,0)$ $B(1,0)$ $C(1,1)$ $\lambda \in (0,1)$
$P(0,\lambda)$ $Q(\lambda,0)$ $p_A^{\perp}(\overrightarrow{PB}) = H$
$H(x_H, y_H)$

1 1)



1,5 2) $\overrightarrow{AH} = \begin{pmatrix} x_H \\ y_H \end{pmatrix}$ $\overrightarrow{PB} = \begin{pmatrix} 1-0 \\ 0-\lambda \end{pmatrix} = \begin{pmatrix} 1 \\ -\lambda \end{pmatrix}$

Thus $\overrightarrow{AH} \cdot \overrightarrow{PB} = 1 \times x_H - \lambda \times y_H = x_H - \lambda y_H$

Since H ~~is~~ is the orthogonal projection of A on (PB), $\overrightarrow{AH}$ and $\overrightarrow{PB}$ are orthogonal and thus $\overrightarrow{AH} \cdot \overrightarrow{PB} = 0$

Hence $x_H - \lambda y_H = 0 \Leftrightarrow \boxed{x_H = \lambda y_H}$

1,5 3) $z=0$ $\overrightarrow{PH}\begin{pmatrix} x_H \\ y_H - \lambda \\ 0 \end{pmatrix}$ $\overrightarrow{HB}\begin{pmatrix} 1 - x_H \\ -y_H \\ 0 \end{pmatrix}$

$$\overrightarrow{PH} \wedge \overrightarrow{HB} = \begin{pmatrix} x_H \\ y_H - \lambda \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 1 - x_H \\ -y_H \\ 0 \end{pmatrix} = \begin{pmatrix} (y_H - \lambda) \times 0 + y_H \times 0 \\ 0 \times (1 - x_H) - 0 \times x_H \\ x_H \times -y_H - (y_H - \lambda) \times (1 - x_H) \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ -y_H x_H - (y_H - x_H y_H - \lambda + \lambda x_H) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\lambda x_H - y_H + \lambda \end{pmatrix}$$

But we also know that $H \in (PB)$, thus $\overrightarrow{PH}$ and $\overrightarrow{HB}$ are colinear, which means that $\overrightarrow{PH} \wedge \overrightarrow{HB} = \vec{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

From our previous result, we deduce that:

$$-\lambda x_H - y_H + \lambda = 0$$

$$\Leftrightarrow \boxed{\lambda = \lambda x_H + y_H}$$

1 4) $\lambda = \lambda x_H + y_H$

$\Leftrightarrow \lambda = \lambda^2 y_H + y_H$ since $x_H = \lambda y_H$.

~~$\Leftrightarrow (\lambda^2 + 1) y_H - \lambda = 0$~~

~~$\Leftrightarrow \lambda^2 y_H - \lambda + y_H = 0$~~

$\Leftrightarrow \lambda = (\lambda^2 + 1) y_H \Leftrightarrow y_H = \dfrac{\lambda}{\lambda^2 + 1}$

Moreover, $x_H = \lambda y_H = \dfrac{\lambda^2}{\lambda^2 + 1}$.

Thus, we have $\boxed{H\begin{pmatrix} \frac{\lambda^2}{\lambda^2+1} \\ \frac{\lambda}{\lambda^2+1} \end{pmatrix}}$

2 5) $\overrightarrow{HC}\begin{pmatrix} 1 - \frac{\lambda^2}{\lambda^2+1} \\ 1 - \frac{\lambda}{\lambda^2+1} \end{pmatrix}$ $\overrightarrow{HQ}\begin{pmatrix} \lambda - \frac{\lambda^2}{\lambda^2+1} \\ \frac{-\lambda}{\lambda^2+1} \end{pmatrix}$

$$\overrightarrow{HC} \cdot \overrightarrow{HQ} = \left(1 - \frac{\lambda^2}{\lambda^2+1}\right)\left(\lambda - \frac{\lambda^2}{\lambda^2+1}\right) + \left(1 - \frac{\lambda}{\lambda^2+1}\right)\left(\frac{-\lambda}{\lambda^2+1}\right)$$

$$= \lambda - \frac{\lambda^2}{\lambda^2+1} - \frac{\lambda^3}{\lambda^2+1} + \frac{\lambda^4}{(\lambda^2+1)^2} - \frac{\lambda}{\lambda^2+1} + \frac{\lambda^2}{(\lambda^2+1)^2}$$

$$= \frac{\lambda(\lambda^2+1)^2 \cancel{- \lambda^4} - \lambda^2 - \lambda^5 - \lambda^3 \cancel{+ \lambda^4} - \lambda^3 - \lambda + \lambda^2}{(\lambda^2+1)^2}$$

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Exercise 6) 1) $\left|\frac{1+ia}{1-ia}\right| = \frac{|1+ia|}{|1-ia|} = \frac{\sqrt{1^2+a^2}}{\sqrt{1^2+(-a)^2}} = \frac{\sqrt{1+a^2}}{\sqrt{1+a^2}} = 1$ ✓

1

1 2)

$\frac{1+ia}{1-ia} = -1 \iff 1+ia = -1+ia \iff 1 = -1$ which is absurd ✓

Thus, we can't have $\frac{1+ia}{1-ia} = -1$. ✓

Hence, $\forall a \in \mathbb{R}$, $\frac{1+ia}{1-ia} \neq -1$. ✓

1 3) $z \neq -1$ $|z| = 1$ $z = a+ib$ $(a, b \in \mathbb{R})$

a) $\overline{\frac{1-z}{1+z}} = \frac{\overline{1-(a+ib)}}{1+a+ib} = \frac{\overline{1-a-ib}}{1+a+ib} = \frac{1-a+ib}{1+a+ib}$

$= \frac{1-a+ib}{1+z}$? ...?

Not finished

If $\overline{\frac{1-z}{1+z}} = -\frac{1-z}{1+z}$?

Then we have $\overline{1-z} = -1-z$

which means that $Re(1-z) = -Re(-1-z)$

$Re(z)=0$

z is imaginary ✓

1,5 b) Since $|z| = 1$, z can be written as $z = e^{i\theta}$ with $\theta \in \mathbb{R}$.

Thus $\frac{1-z}{1+z} = \frac{1-e^{i\theta}}{1+e^{i\theta}} = \frac{\cancel{e^{\frac{i\theta}{2}}}\left(e^{-\frac{i\theta}{2}} - e^{\frac{i\theta}{2}}\right)}{\cancel{e^{\frac{i\theta}{2}}}\left(e^{-\frac{i\theta}{2}} + e^{\frac{i\theta}{2}}\right)} = \frac{-2i\sin\left(\frac{\theta}{2}\right)}{2\cos\left(\frac{\theta}{2}\right)}$

$= -2i\tan\left(\frac{\theta}{2}\right)$ ✓