MILAZZO
Etienne
SCAN 62

MTES Test n°2

January 1[illegible]
2019

20,00

Exercise 2) Define $\omega = df$

$$\omega = \frac{yz + xy + 1}{z + x}\,dx + x\,dy + \frac{1}{x+z}\,dz$$

2

1) $\frac{d^2 f}{dx\,dy} = 1$ $\qquad \frac{d^2 f}{dy\,dx} = \frac{x+z}{x+z} = 1$

$\frac{d^2 f}{dz\,dy} = 0$ $\qquad \frac{d^2 f}{dy\,dz} = 0$

$\frac{d^2 f}{dx\,dz} = \frac{-1}{(x+z)^2}$ $\qquad \frac{d^2 f}{dz\,dx} = \frac{y(z+x) - 1\times(yz + xy + 1}{(x+z)^2}$

$= \frac{xy + zy - zy - xy - 1}{(x+z)^2} = \frac{-1}{(x+z}$

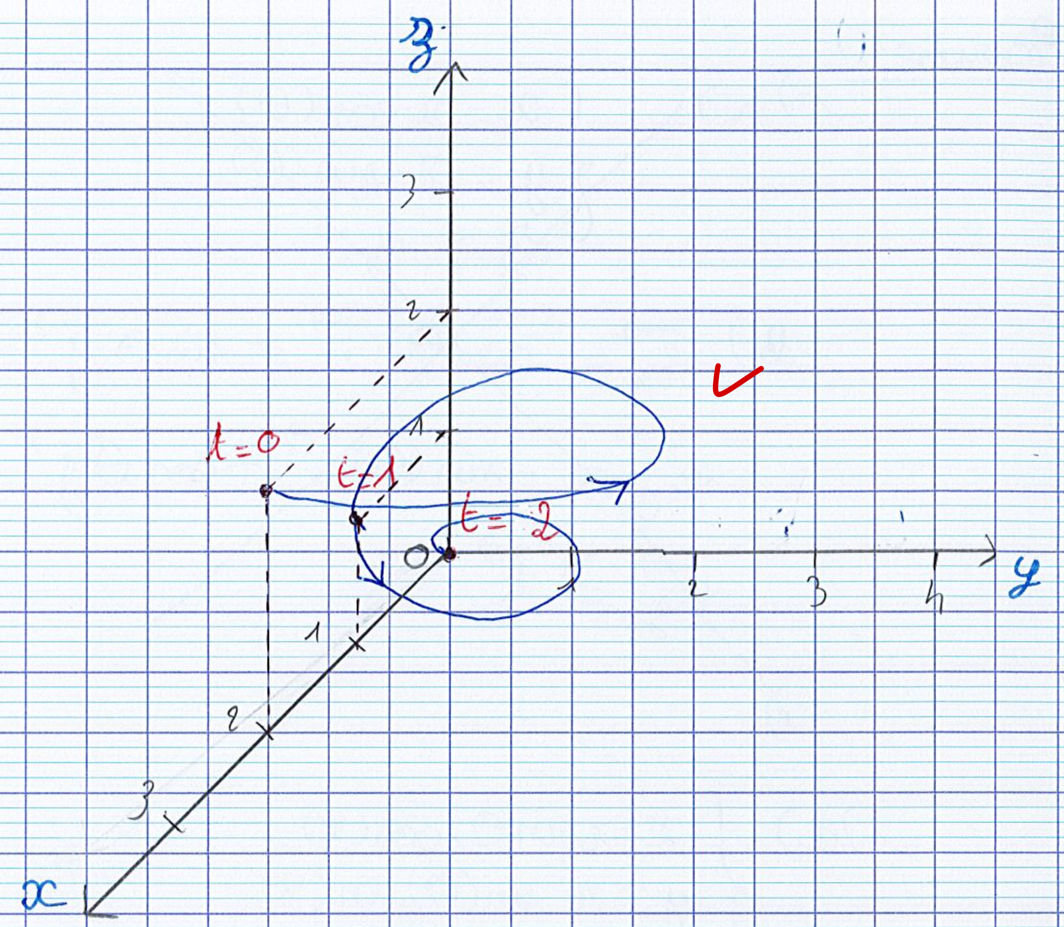
Thus, we have: $\frac{d^2 f}{dx\,dy} = \frac{d^2 f}{dy\,dx}$

$\frac{d^2 f}{dz\,dy} = \frac{d^2 f}{dy\,dz}$

$\frac{d^2 f}{dx\,dz} = \frac{d^2 f}{dz\,dx}$

Thus, ω is a closed form.

1,25 b)



0,5 c) $\vec{e_r}(t) = \sin(\theta)\cos(\varphi)\,\vec{i} + \sin(\theta)\sin(\varphi)\,\vec{j} + \cos(\theta)\,\vec{k}$

$= \sin\left(\frac{\pi}{4}\right)\cos(2\pi t)\,\vec{i} + \sin\left(\frac{\pi}{4}\right)\sin(2\pi t)\,\vec{j} + \cos\left(\frac{\pi}{4}\right)\vec{k}$

$= \frac{\sqrt{2}\cos(2\pi t)}{2}\,\vec{i} + \frac{\sqrt{2}\sin(2\pi t)}{2}\,\vec{j} + \frac{\sqrt{2}}{2}\,\vec{k}$

1 d) $\frac{d\vec{e_r}}{dt} = \frac{-\sqrt{2}}{2} \times 2\pi \sin(2\pi t)\,\vec{i} + \frac{\sqrt{2}}{2} \times 2\pi\cos(2\pi t)\,\vec{j}$

$= -\sqrt{2}\,\pi \sin(2\pi t)\,\vec{i} + \sqrt{2}\,\pi\cos(2\pi t)\,\vec{j}$

$= \sqrt{2}\,\pi\left(-\sin(2\pi t)\,\vec{i} + \cos(2\pi t)\,\vec{j}\right)$

$= \sqrt{2}\,\pi\left(-\sin(\varphi(t))\,\vec{i} + \cos(\varphi(t))\,\vec{j}\right)$

$= \sqrt{2}\,\pi\,\vec{e_\varphi}$

1,5 2) condition?

Integrate df by y:

$$f = xy + g(x) + h(z)$$

Differentiate by z:

$$\frac{df}{dz} = 0 + 0 + h'(z) \quad \text{, hence } h'(z) = \frac{1}{x+z}$$

Hence $h(z) = \ln(x+z)$

Integrate by z:

$$f = xy + \ln(x+z) + h(z)$$

Differentiate by x:

$$\frac{df}{dx} = y + \frac{1}{x+z} + h'(z) = \frac{yx+yz+1}{x+z} + h'(z)$$

Hence $h'(z) = 0$

hence $h(z) = C$, $C \in \mathbb{R}$

Hence $f(x,y,z) = xy + \ln(x+z) + C$

→ this is a consequence!

Thus there exists a function f defined on D such that $df = \omega$, hence we can conclude ω is an exact form. ×

Exercice 1)

1) a) $\begin{cases} x = r\cos(\theta) \\ y = r\sin(\theta) \\ z = z \end{cases}$

b) $\vec{e_r} = \cos(\theta)\vec{i} + \sin\theta\,\vec{j}$

$\vec{e_\theta} = -\sin(\theta)\vec{i} + \cos\theta\,\vec{j}$

$\vec{e_z} = \vec{k}$

c)

1,5 1) a) $\begin{cases} x = r\sin(\theta)\cos(\varphi) \\ y = r\sin(\theta)\sin(\varphi) \\ z = r\cos(\theta) \end{cases}$

1,5 b) $\vec{e_r} = \sin(\theta)\cos(\varphi)\vec{i} + \sin(\theta)\sin(\varphi)\vec{j} + \cos(\theta)\vec{k}$

$\vec{e_\theta} = \cos(\theta)\cos(\varphi)\vec{i} + \cos(\theta)\sin(\varphi)\vec{j} - \sin(\theta)\vec{k}$

$\vec{e_\varphi} = -\sin(\varphi)\vec{i} + \cos(\varphi)\vec{j}$

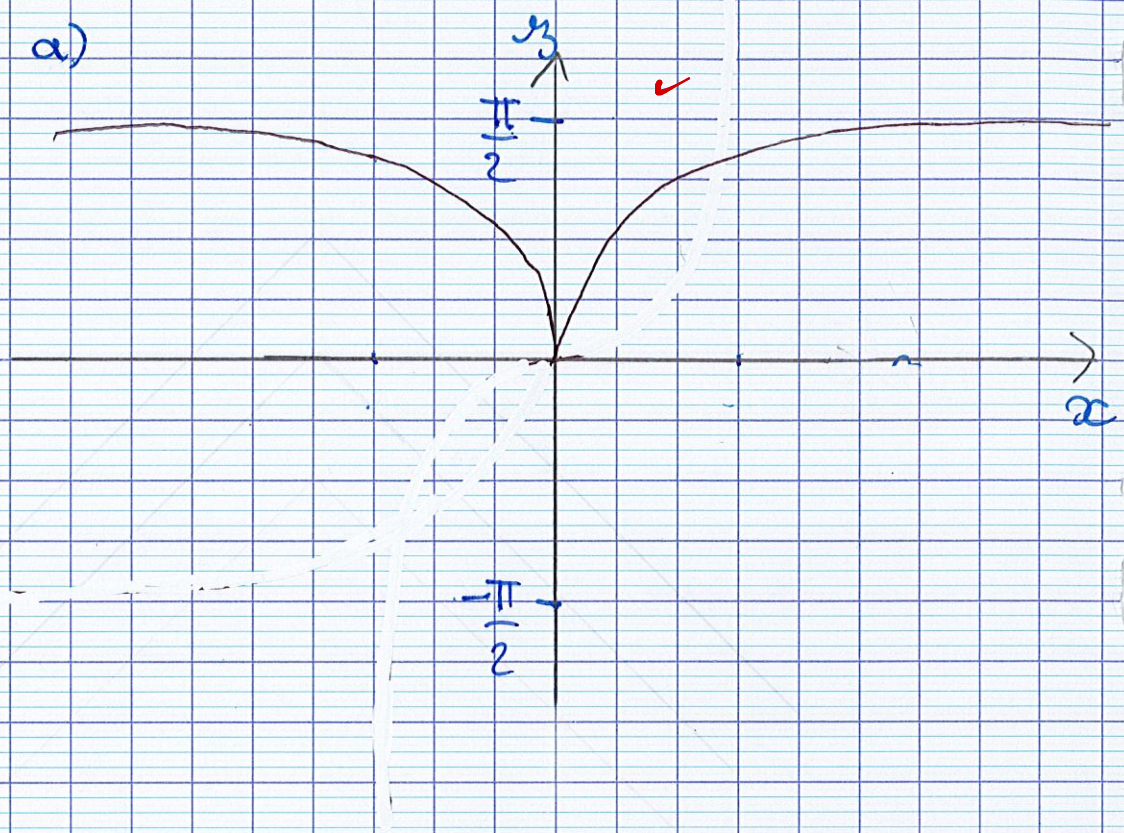
0,5 c) $\vec{OM} = r\,\vec{e_r}$

2) $\begin{cases} r(t) = (2-t)\sqrt{2} \\ \theta(t) = \frac{\pi}{4} \\ \varphi(t) = 2\pi t \end{cases}$ $t \in [0, 2]$

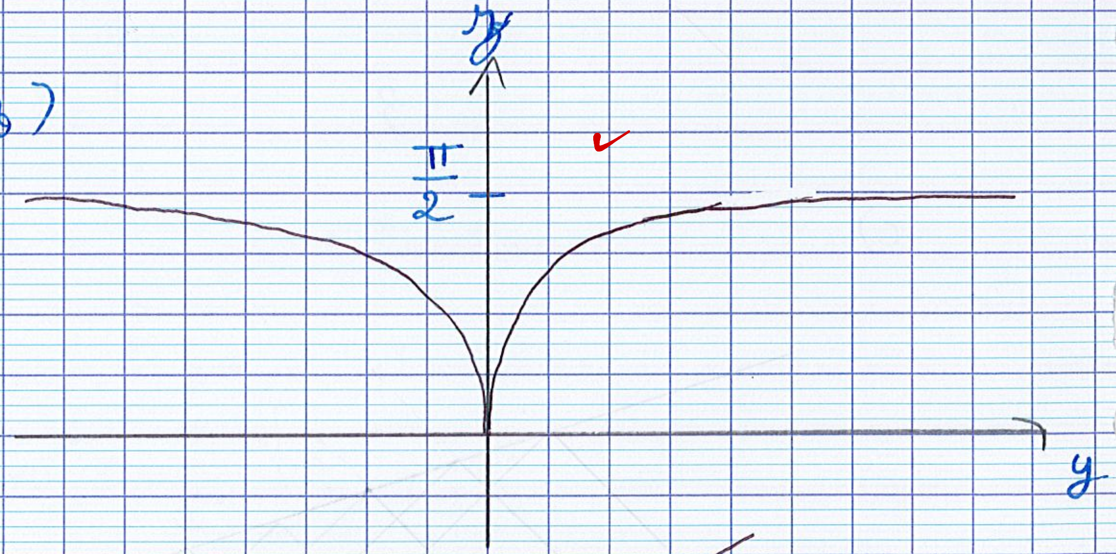
0,5 a) Since θ is constant, this trajectory is on a cone. missing more details on the cone shape!

2)

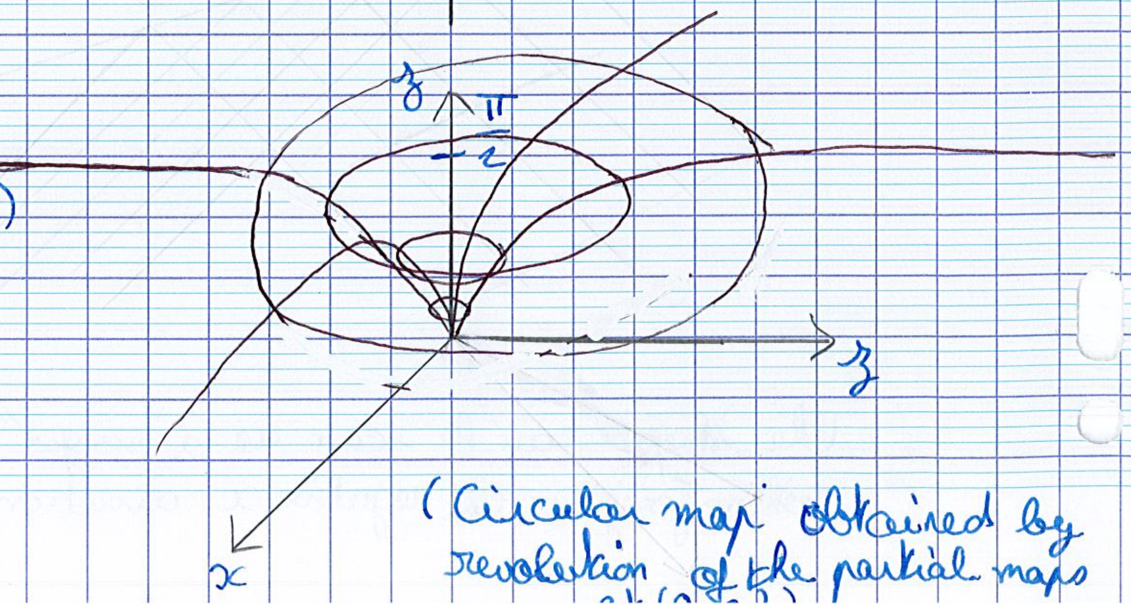
0,5 a)



0,25 b)



1 c)



(Circular map obtained by revolution of the partial maps

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1 e) $\vec{v}(t) = \overrightarrow{OM}'(t) = \dfrac{d\,\overrightarrow{OM}}{dt} = \dfrac{d(r\,\vec{e_r})}{dt}$

$= \dfrac{dr}{dt}\,\vec{e_r}(t) + \dfrac{d\vec{e_r}}{dt}\,r(t)$

$= -\sqrt{2}\,\vec{e_r}(t) + \sqrt{2}\pi\,\vec{e_\varphi}(t)(2-t)\sqrt{2}$

$= -\sqrt{2}\,\vec{e_r}(t) + 2\pi(2-t)\,\vec{e_\varphi}(t)$

(with $\vec{e_r}$ and $\vec{e_\varphi}$ defined previously)

Exercise 3) $C = k\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)^{-1} = \dfrac{k R_1 R_2}{R_2 - R_1}$

3 1) $\delta C = \dfrac{R_1 R_2}{R_2 - R_1}\,\delta k + \dfrac{k R_2 (R_2 - R_1) - k R_1 R_2 \times -1}{(R_2 - R_1)^2}$

$+ \dfrac{k R_1 (R_2 - R_1) - k R_1 R_2 \times 1}{(R_2 - R_1)^2}\,\delta R_2$

$\delta C = \dfrac{R_1 R_2}{R_2 - R_1}\,\delta k + \dfrac{k R_2^2}{(R_2 - R_1)^2}\,\delta R_1 - \dfrac{k R_1^2}{(R_2 - R_1)^2}\,\delta$

Hence (N.A.):

$\delta C = \dfrac{1 \times 2}{2-1} \times 10^{-3} + \dfrac{1 \times 2^2}{(2-1)^2}\,\delta R_1 - \dfrac{1 \times 1^2}{(2-1)^2} \times 4\,\delta R_1$

$\delta C = 0{,}002 + 4\delta R_1 - 4\delta R_1 = 0{,}002$ units?

2 2) $\Delta C = \left|\frac{R_1 R_2}{R_2 - R_1}\right| \Delta k + \left|\frac{k R_2^2}{(R_2 - R_1)^2}\right| \Delta R_1 + \left|\frac{k R_1^2}{(R_2 - R_1)^2}\right| \Delta R_2$

Thus, (N.A.), we have:

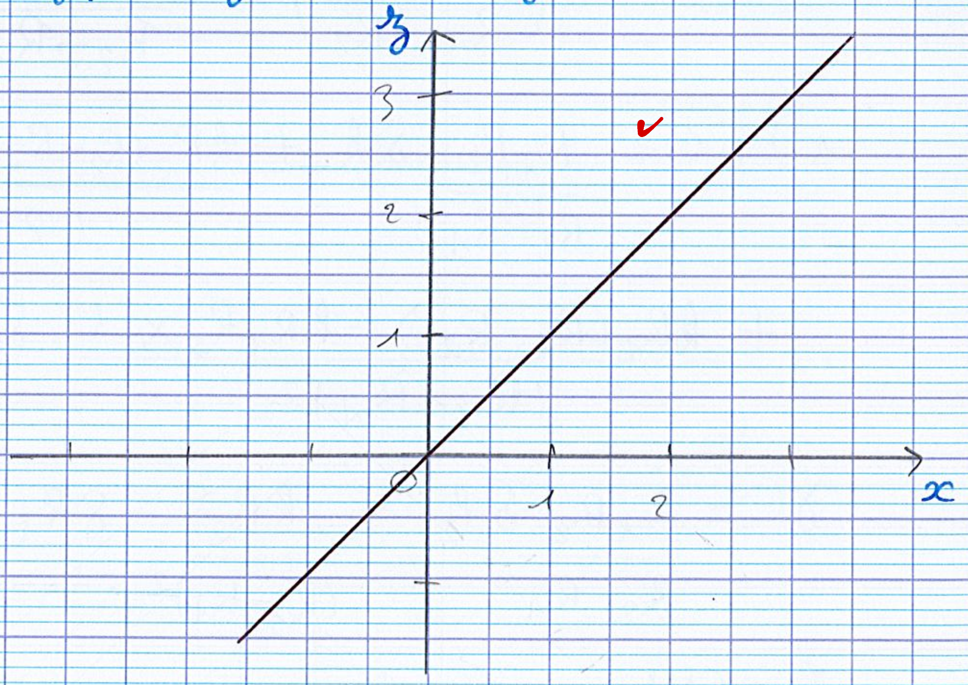
$$\Delta C = \left|\frac{1\times 2}{2-1}\right| 10^{-3} + \left|\frac{1\times 2^2}{(2-1)^2}\right| 10^{-3} + \left|\frac{1\times 1^2}{(2-1)^2}\right| 10^{-3}$$

$$\Delta C = 2\times 10^{-3} + 4\times 10^{-3} + 1\times 10^{-3}$$

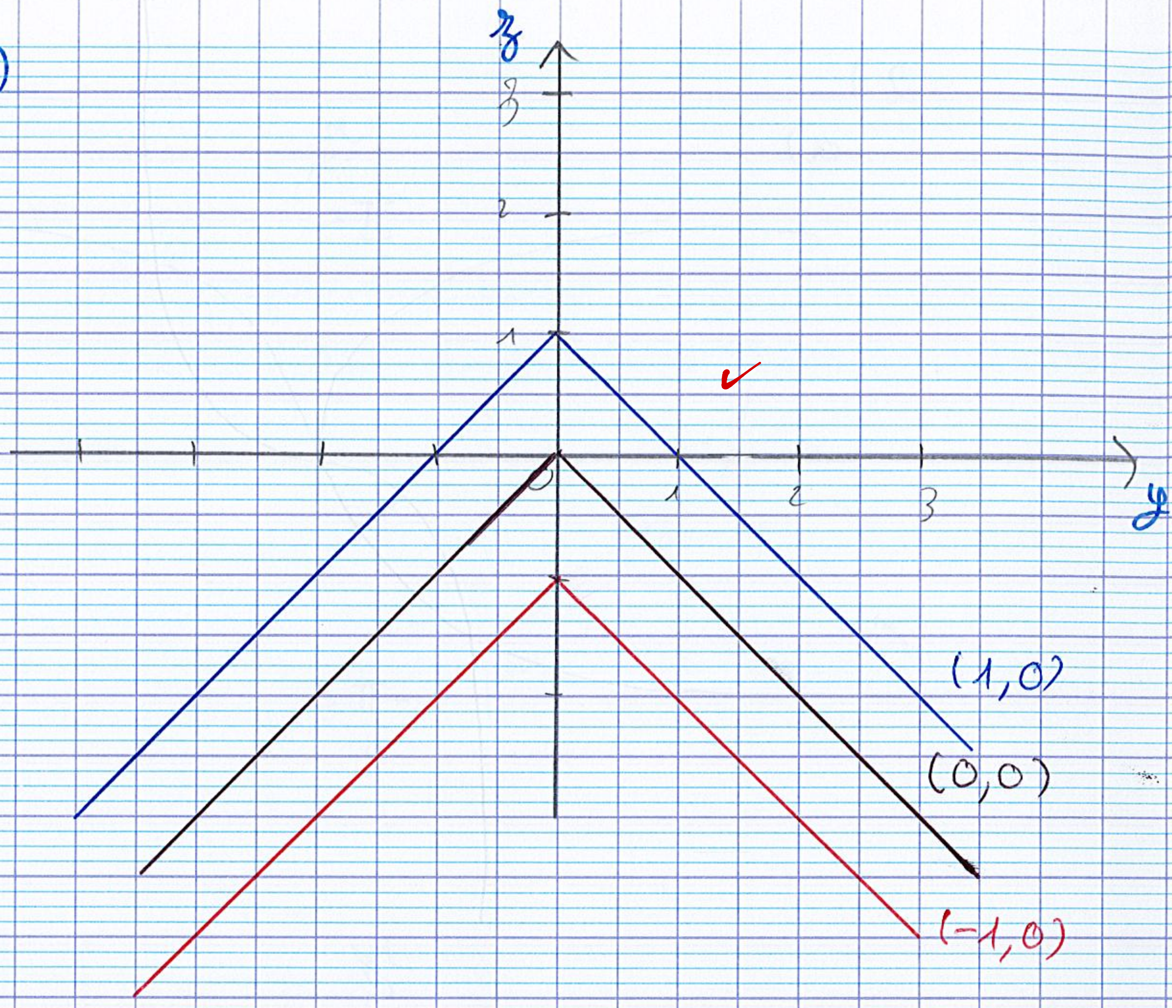
$\Delta C = 0{,}007$ units?

Exercise 1) 1) $f_1 : (x,y) \longmapsto x - |y|$

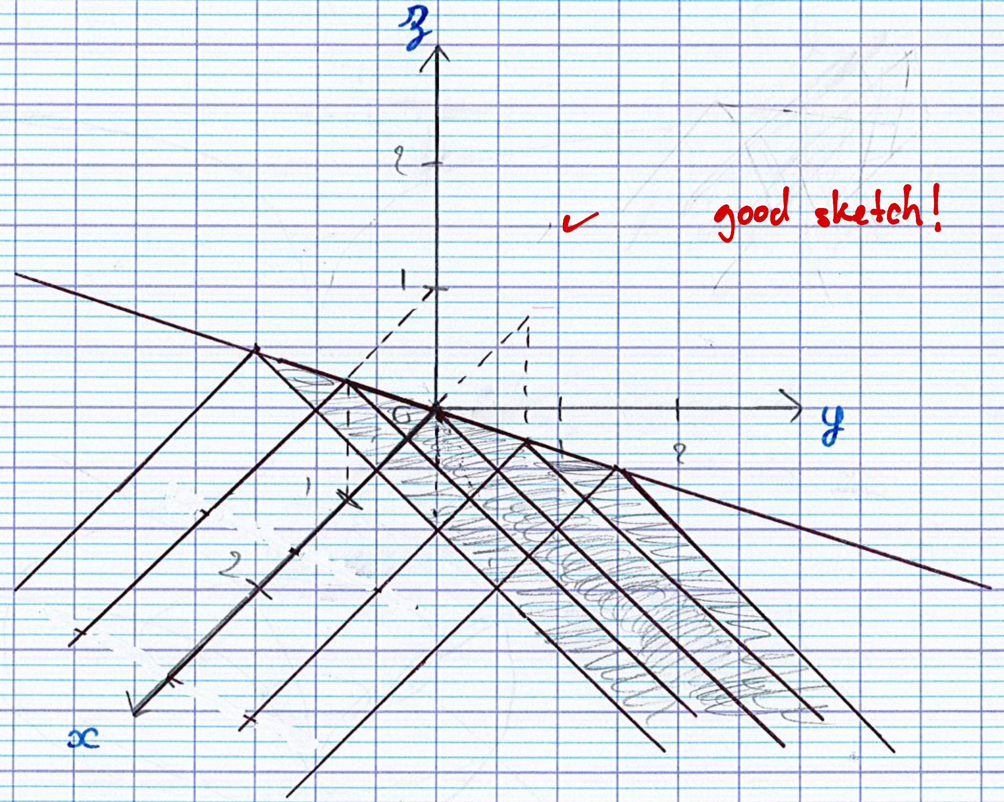
0,25 a)



0,75 b)



1 c) good sketch!



The shape can be seen as a "roof", descending when going in the negative x direction.