INSTITUT NATIONAL DES SCIENCES APPLIQUÉES DE LYON

Département du Premier Cycle - SCAN - First

IE 2 MTES - Duration 1h30

Warnings and advices
- **Documents, dictionaries, phones, and calculators are FORBIDDEN.**
- **The marking scheme is only given as an indication.**

EXERCISE 1 *(4 pts)*

No justifications required !

Consider the following maps f_1 and f_2 defined below from $\mathbb{R}^2$ to $\mathbb{R}$ by :
- $f_1 : (x, y) \mapsto x - |y|$
- $f_2 : (x, y) \mapsto arctan(\sqrt{x^2 + y^2})$

1. For f_1, draw the graphs of :
   (a) The partial map with respect to x at the points $(0, 0)$.
   (b) The partial maps with respect to y at the points $(-1, 0)$, $(0, 0)$ and $(1, 0)$. (On the same picture)
   (c) The map f_1 from $\mathbb{R}^2$ to $\mathbb{R}$.
2. For f_2, draw the graphs of :
   (a) The partial map with respect to x at the points $(0, 0)$.
   (b) The partial map with respect to y at the points $(0, 0)$.
   (c) The map f_2 from $\mathbb{R}^2$ to $\mathbb{R}$.

EXERCISE 2 *(4 pts)*

We consider the differential form $\omega = \dfrac{yz + xy + 1}{z + x}\,\mathrm{d}x + x\,\mathrm{d}y + \dfrac{1}{x + z}\,\mathrm{d}z$ on $D = \{(x, y, z) \in \mathbb{R}^3 | x > 0, y > 0, z > 0\}$.

1. Show that the differential form ω is a closed form on D.
2. Is it an exact form on D? If yes, give a function f defined on D such that $df = \omega$.

EXERCISE 3 *(5 pts)*

The capacitance C of a spherical capacitor is given by the formula $C = k\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)^{-1}$ where k is a positive constant in $F.m^{-1}$ that depends on the capacitor and R_1 and R_2 are the radii with $R_1 < R_2$.

1. Over time, the values of k, R_1 and R_2 are modified of δk, δR_1 and δR_2.
   Express the variation δC with respect to the variations δk, δR_1 and δR_2.
   N.A. : $k = 1F.m^{-1}$, $R_1 = 1m$, $R_2 = 2m$, $\delta k = 10^{-3}F.m^{-1}$, $\delta R_2 = 4\delta R_1$
2. The manufacturer can give the values of k, R_1 and R_2 with uncertainties of Δk, ΔR_1 and ΔR_2.
   Express the uncertainty ΔC with respect to the uncertainties Δk, ΔR_1 and ΔR_2.
   N.A. : $k = 1F.m^{-1}$, $R_1 = 1m$, $R_2 = 2m$, $\Delta k = 10^{-3}F.m^{-1}$, $\Delta R_2 = \Delta R_1 = 10^{-3}m$.

EXERCISE 4 *(7 pts)*

No justifications are required in this exercise except for the last 2 questions !

1. (a) Recall the expression of the cartesian coordinates x, y, z with respect to the spherical coordinates r, θ, φ.

   (b) Recall the expression of the vectors $\overrightarrow{e_r}, \overrightarrow{e_\theta}$ and $\overrightarrow{e_\varphi}$ of the local spherical frame with respect to the cartesian vectors $\overrightarrow{i}$, $\overrightarrow{j}$ and $\overrightarrow{k}$.

   (c) Recall the expression of the position vector $\overrightarrow{OM}$ in the spherical frame with spherical coordinates.

2. Consider the following parametric curve in spherical coordinates :

$$\begin{cases} r(t) &= (2-t)\sqrt{2} \\ \theta(t) &= \dfrac{\pi}{4} \\ \varphi(t) &= 2\pi t \end{cases} \qquad t \in [0, 2]$$

   It represents the trajectory of a point $M(t)$.

   (a) On what surface is this trajectory ?

   (b) Give a representation of the trajectory of $M(t)$. Place the points for $t = 0$, $t = 1$ and $t = 2$.

   (c) Give the expression of $\overrightarrow{e_r}(t)$ on this parametric curve.

   (d) Show that on this parametric curve $\dfrac{d\overrightarrow{e_r}(t)}{dt} = \sqrt{2}\pi\overrightarrow{e_\varphi}$.

   (e) Compute the velocity $\overrightarrow{v}(t) = \overrightarrow{OM}'(t)$ in the spherical frame.