INSTITUT NATIONAL DES SCIENCES APPLIQUÉES DE LYON

Département du Premier Cycle - SCAN - First

IE 1- 03/05/19 MTES - Duration 1h30

Warnings and advices

- **Documents, dictionaries, phones, and calculators are FORBIDDEN.**
- **The marking scheme is only given as an indication.**

EXERCISE 1 *(3 pts)*

Consider the plane $\mathbb{R}^2$ with an orthonormal frame $(O, \overrightarrow{i}, \overrightarrow{j})$.

Let T be the triangle OAB with the points $A(2,0)$ and $B(2,1)$.

1. Show that in **cartesian coordinates**, the domain T is simple by giving the 2 possible descriptions.
2. Using the previous question, compute the integral $\displaystyle\int_{y=0}^{1}\int_{x=2y}^{2} e^{x^2}\, dx\, dy$.

EXERCISE 2 *(3 pts)*

Consider the domain D of $\mathbb{R}^3$

$$D = \{(x,y,z) \in \mathbb{R}^3 | x \geqslant 0, y \geqslant 0, z \geqslant 0, z \leqslant y, x^2 + y \leqslant 1\}$$

1. Let $(O, \overrightarrow{i}, \overrightarrow{j}, \overrightarrow{k})$ be an orthonormal frame of $\mathbb{R}^3$. Don't try to represent D entirely, just the parts of D in the planes Oxy, Oxz and Oyz.
2. Compute the integral $\displaystyle\iiint_D x\,y\,z\, dx\, dy\, dz$ with a summation by slice on x. What is the shape of the slices ?

EXERCISE 3 *(5 pts)*

Consider a bowl in $\mathbb{R}^3$ of equation $r(z) = 2\sqrt{z}$ for $z \in [0,4]$ **in cylindrical coordinates.**

1. Draw a representation of this bowl.
2. Compute the lateral area of this bowl.
3. Compute the volume of liquid this bowl can contain.
4. The bowl is filled with an homogeneous liquid. Compute the coordinates of the center of mass of the liquid.

EXERCISE 4 *(9 pts)*

Consider the plane $\mathbb{R}^2$ with an orthonormal frame $(O, \vec{i}, \vec{j})$.
We consider the curve C called the cardioid given in polar coordinates by the equation $r = 1+\cos(\theta)$ with $\theta \in [-\pi, \pi]$.

1. Sketch the graph of this curve. To help you, place the points for $\theta = 0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}$ and π.
2. Compute the area of the surface inside this curve.
3. In polar coordinates, we consider the plate

$$S = \{(r, \theta) | -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}, 0 \leqslant r \leqslant 1 + \cos(\theta)\}$$

   whose area density is given by $\sigma = \dfrac{1}{(1+\cos(\theta))^2}$.

   (a) Represent this plate.
   (b) Compute the mass of this plate.
   (c) Determine the coordinates of the center of mass of S.
4. In spherical coordinates in $\mathbb{R}^3$, we consider the homogeneous solid

$$V = \{(r, \theta, \varphi) | 0 \leqslant r \leqslant 1 + \cos(\theta), 0 \leqslant \theta \leqslant \pi, 0 \leqslant \varphi \leqslant 2\pi\}$$

   (a) Justify in one sentence that the solid has a symmetry of revolution around the Oz axis.
   (b) Try to represent the solid V. How does one get V from C?
   (c) Compute the volume of V.