INSTITUT NATIONAL DES SCIENCES APPLIQUÉES DE LYON

SCAN-1
MTES - Semester 1

2019-2020

TEST 1 - 15/11/2019 - 1H

Documents, dictionaries, phones, and calculators are FORBIDDEN.

EXERCISE 1 *(4 pts)*
Solve in $\mathbb{C}$ the equation : $z^2-(4-3\mathrm{i})z+1-7\mathrm{i}=0$.

EXERCISE 2 *(4 pts)*
Let $\theta \in (\frac{\pi}{2}, \frac{3\pi}{2})$. Consider the complex number $Z=\mathrm{i}+\tan(\theta)$. Give the modulus of z and an argument of z.

EXERCISE 3 *(3 pts)*
Let A, B, M three points of $\mathbb{R}^2$. And let I be the middle of $[AB]$.

1. Show that $\overrightarrow{MA}\cdot\overrightarrow{MB}=MI^2-\dfrac{AB^2}{4}$
2. Let $k\in\mathbb{R}$. Regarding the value of k, describe the geometrical set of points of $\mathbb{R}^2$ satisfying the equation $\overrightarrow{MA}\cdot\overrightarrow{MB}=k$.

EXERCISE 4 *(2 pts)*
Consider the points $A(1,1)$, $B(1,-3)$ and $C(1,2)$ in $\mathbb{R}^2$. After having justified its existence, compute the coordinates of the barycenter G of the weighted points $(A,-2),(B,3)$ and $(C,1)$.

EXERCISE 5 *(7 pts)*
Let $\overrightarrow{u}, \overrightarrow{v}, \overrightarrow{w}$ be 3 vectors of $\mathbb{R}^3$. And let $\overrightarrow{s}=\overrightarrow{u}\wedge(\overrightarrow{v}\wedge\overrightarrow{w})$.

1. **In this question only**, we take $\overrightarrow{u}=(1,1,1)$, $\overrightarrow{v}=(0,1,0)$ and $\overrightarrow{w}=(0,0,1)$. Compute
   (a) $((\overrightarrow{u},\overrightarrow{v},\overrightarrow{w}))$.
   (b) The projection of $\overrightarrow{v}$ on the straight line D directed by $\overrightarrow{u}$.
   (c) $\overrightarrow{s}$.
2. Compute $\overrightarrow{s}$ if $\overrightarrow{v}$ and $\overrightarrow{w}$ are collinear.
3. Compute $\overrightarrow{s}$ if $\overrightarrow{u}$ is orthogonal to both $\overrightarrow{v}$ and $\overrightarrow{w}$.
   In the rest of the exercise, we assume that $\overrightarrow{v}$ and $\overrightarrow{w}$ are not collinear.
4. Justify that $\overrightarrow{s}$ is in the plane generated by $\overrightarrow{v}$ and $\overrightarrow{w}$.
5. The previous result implies that there exists $\alpha,\beta\in\mathbb{R}$ such that : $\overrightarrow{s}=\alpha\overrightarrow{v}+\beta\overrightarrow{w}$.
   (a) Show that $0=\alpha(\overrightarrow{v}\cdot\overrightarrow{u})+\beta(\overrightarrow{w}\cdot\overrightarrow{u})$.
   (b) Show that $\overrightarrow{s}\cdot\overrightarrow{v}=((\overrightarrow{v}\wedge\overrightarrow{w},\overrightarrow{v},\overrightarrow{u}))$.
   (c) Deduce from it that $\alpha\|\overrightarrow{v}\|^2+\beta(\overrightarrow{v}\cdot\overrightarrow{w})=(\overrightarrow{v}\wedge\overrightarrow{w})\cdot(\overrightarrow{v}\wedge\overrightarrow{u})$.