INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

Scan First
Year 2020-2021

MTES – Correction of the exam #1

Exercise 1 (4 points)

	$1-i\sqrt{3}=2e^{-i\pi/3}$	0.5 modulus + 0.5 arg
	$1-i=\sqrt{2}e^{-i\pi/4}$	0.5 modulus + 0.5 arg
	$\frac{(1-i\sqrt{3})^5}{(1-i)^3}=2^{7/2}.e^{-i11\pi/12}$	0.5 modulus + 0.5 arg
	$\left(\frac{(1-i\sqrt{3})^5}{(1-i)^3}\right)^n$ is positive real iif its argument is a multiple of 2π	0.5
	i.e. n is a multiple of 24/11	0.5

Exercise 2 (5.5 points)

1	The equation is $z^2+z+(1-i)=0$	0.25
	$\Delta=-3+4i$	0.5
	$\Delta=\delta^2$ with $\delta=a+ib$ gives $\begin{cases} a^2-b^2=-3 \\ 2ab=4 \\ a^2+b^2=5 \end{cases}$	3*0.25
	We keep $\delta=1+2i$	0.5
	Hence $z_1=-1-i$ and $z_2=i$	2*0.5
2	$z_1=z_2$ if $\Delta=0$	0.5
	We get $\omega=2$	0.5
3	M, M_1, M_2 are aligned iff $\left(\widehat{\overrightarrow{M_1M_2},\overrightarrow{M_1M}}\right)=Arg\left(\frac{\omega-z_1}{z_2-z_1}\right)=0$	0.5
	it means $\frac{\omega-z_1}{z_2-z_1}$ is a real	0.5
	therefore $\frac{\omega-z_1}{z_2-z_1}=\frac{\bar{\omega}-\bar{z}_1}{\bar{z}_2-\bar{z}_1}$	0.5

Exercise 3 (4.5 points)

1	$\overrightarrow{AB}\,(0,-3,0)$; $\overrightarrow{AC}\,(-1,-1,1)$; $\overrightarrow{AD}\,(1,1,-1)$;	3*0.5
2	$\overrightarrow{AB}\wedge\overrightarrow{AD}\,(3,0,3)$	1
	$\left(\left(\overrightarrow{AB},\overrightarrow{AC},\overrightarrow{AD}\right)\right)=0$	1
3	the vectors $\overrightarrow{AB},\overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar because $\left(\left(\overrightarrow{AB},\overrightarrow{AC},\overrightarrow{AD}\right)\right)=0$ or because $\overrightarrow{AC}$ and $\overrightarrow{AD}$ are collinear	0.5 conclusion + 0.5 explanation

Exercise 4 (7 points)

1	$\vec{D} \perp \vec{u}$ since $\vec{u}.\vec{D} = 0$ $\vec{B} \perp \vec{u}$ since $\vec{u}.\vec{B} = 0$ $\vec{H} \perp \vec{u}$ since $\vec{u} \wedge \vec{E} = \mu c\vec{H}$	0.25+0.25 0.25+0.25 0.25+0.25
2	$\vec{E},\ \vec{H} \perp \vec{R}$ since $\vec{E} \wedge \vec{H} = \vec{R}$ $\vec{B} \perp \vec{R}$ since $\vec{B} \wedge \vec{H} = \vec{0}$ implies $\vec{B}$ and $\vec{H}$ are collinear	2*0.25+0.25 0.25+0.5
3	$\vec{H} \perp \vec{u}$, cf question 1 $\vec{H} \perp \vec{R}$, cf question 2 $\vec{H} \perp \vec{D}$ from $\vec{u} \wedge \vec{H} = -c\vec{D}$ $\vec{H} \perp \vec{E}$ from $\vec{u} \wedge \vec{E} = \mu c\vec{H}$	0.25 0.25 0.25+0.25 0.25+0.25
4	From question 3, $\vec{u}, \vec{D}, \vec{E}, \vec{R}$ are coplanar This plane is perpendicular to $\vec{H}, \vec{B}$	0.25 0.25
5	If $\vec{u}$ and $\vec{E}$ were collinear, $\vec{u} \wedge \vec{E}$ would be nil Here we have $\vec{u} \wedge \vec{E} = \mu c\vec{H} \neq \vec{0}$	0.5 0.5
6	$\left((\vec{u}, \vec{E}, \vec{B})\right) = \left((\vec{B}, \vec{u}, \vec{E})\right) = \vec{B}.(\vec{u} \wedge \vec{E}) = \vec{B}.\mu c\vec{H} = \mu c\vec{B}.\vec{H} > 0$	0.5 permutation 0.5 result