INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

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Year 2020-2021

MTES – Exam #1

November 9, 2020 **Duration: 1 h**

No document allowed. No mobile phone. ***No calculator allowed****. The proposed grading scale is only indicative.*

Exercise 1 (≈ 4 points)

Determine the values of n for which the complex number $\left(\frac{(1-i\sqrt{3})^5}{(1-i)^3}\right)^n$ is a positive real.

Exercise 2 (≈ 5 points)

Let consider ω a complex number, and the following equation with one unknown z :

$$z^2 + (2 + i\omega)z + (i\omega + 2 - \omega) = 0$$

The solutions are called z_1 and z_2.

1. Give the expression of z_1 and z_2 in the case $\omega = i$.
2. Which value of ω implies $z_1 = z_2$?
3. Let M, M_1, M_2 be points in $\mathbb{R}^2$, of respective affixes ω, z_1 and z_2 (in the case $z_1 \neq z_2$). Admitting that $\left(\widehat{\overrightarrow{M_1M_2}, \overrightarrow{M_1M}}\right) = Arg\left(\frac{\omega - z_1}{z_2 - z_1}\right)$, prove that M, M_1, M_2 are aligned iff $(\omega - z_1)(\overline{z_2} - \overline{z_1}) = (\overline{\omega} - \overline{z_1})(z_2 - z_1)$.

Exercise 3 (≈ 4 points)

Consider the points A (1,2,1), B (1,-1,1), C (0,1,2) and D (2,3,0) in $\mathbb{R}^3$.

1. Compute the coordinates of vectors $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$.
2. Compute $\overrightarrow{AB} \wedge \overrightarrow{AD}$ and $\left(\left(\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}\right)\right)$.
3. What can you conclude about the vectors $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$?

Exercise 4 (≈ 7 points)

In physics, the propagation of electromagnetic waves obeys several laws. We consider the following relations, where c and μ are **strictly positive constants** and $\vec{u}, \vec{D}, \vec{E}, \vec{H}, \vec{B}, \vec{R}$ are 6 **non-zero vectors** in $\mathbb{R}^3$:

$\vec{u}.\vec{D} = 0$ $\quad \vec{B} \wedge \vec{H} = \vec{0}$ $\quad \vec{u} \wedge \vec{H} = -c\vec{D}$ $\quad \vec{H}.\vec{B} > 0$

$\vec{u}.\vec{B} = 0$ $\quad \vec{E} \wedge \vec{H} = \vec{R}$ $\quad \vec{u} \wedge \vec{E} = \mu c\vec{H}$

1. Cite 3 vectors that are perpendicular to $\vec{u}$. Each time, give a short explanation.
2. Cite 3 vectors that are perpendicular to $\vec{R}$. Each time, give a short explanation.
3. Cite 4 vectors that are perpendicular to $\vec{H}$. Each time, give a short explanation.
4. Deduce that 4 vectors out of 6 belong to the same plane, and that this plane is perpendicular to the 2 other vectors (indicate which ones).
5. Prove that $\vec{u}$ and $\vec{E}$ are not collinear.
6. Give the sign of the scalar triple product $\left(\left(\vec{u}, \vec{E}, \vec{B}\right)\right)$.