INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

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Year 2020-2021

MTES – Exam #1

January 15, 2021 **Duration: 1.5 h**

No document allowed. No mobile phone. ***Any type of calculator allowed.*** *The proposed grading scale is only indicative.*

Exercise 1 (≈ 3 points)

A tank, having the shape of a vertical cylinder of radius R and height H, is filled with a volume V of water. We want to study the height h of water inside the reservoir : $h = \frac{V}{\pi R^2}$.

1. Because of variations of the temperature, the tank radius varies by δR. Water evaporation or condensation induce changes of the water volume by δV.
   a. Considering δR and δV are small, give an approximation of the relative variation of the water height $\frac{\delta h}{h}$.
   b. Give the value of $\frac{\delta h}{h}$ for $R = 1\,m$; $\delta R = 1\,mm$; $V = 10\,m^3$ and $\frac{\delta V}{V} = 0.6\,\%$.

2. R and H are given with the absolute uncertainties ΔR and ΔV, respectively.
   a. Give the literal expression of the relative uncertainty on h, namely $\frac{\Delta h}{h}$.
   b. Give the value of $\frac{\Delta h}{h}$ for $R = 1\,m$; $\Delta R = 1\,mm$; $V = 10\,m^3$ and $\frac{\Delta V}{V} = 1\,\%$.
   c. Express the height as $h = (\ldots \pm \ldots)$ unit.

Exercise 2 (≈ 2 points)

During a physics lab on chromatic aberrations, you measured the focal length f' of a converging lens using the Bessel's method. With L the distance between the object and the screen ($L > 4f'$) and l the distance between both lens positions for which a sharp image is obtained on the screen, we get:

$$f' = \frac{L^2 - l^2}{4L}$$

L and l are measured with an absolute uncertainty $\Delta L = \Delta l = \varepsilon$. We know that $0 < \varepsilon \ll l < L$.

Determine the absolute uncertainty on f' as a function of L, l and ε.

Exercise 3 (≈ 4 points)

We consider the differential form ω on $D = \{(x, y) \in \mathbb{R}^2 |\, y > 0\}$:

$$\omega = \left(\frac{y}{2} + 1\right).dx - \frac{x}{y}.dy$$

1. Is ω a closed-form?
2. We consider ϕ a C^1-function on $]0; +\infty[$ and we define the differential form Ω as $\Omega = \phi(y).\omega$
   a. Under which condition is Ω a closed-form? The condition may be expressed as a function of x, y, $\phi(y)$ and its derivative $\phi'(y)$.

b. Deduce that Ω is closed if and only if $\phi(y) = \frac{K}{y}$ where K is a real constant.

c. With $K = 1$, find all the functions $f(x, y)$ such that $\Omega = df$.

Exercise 4 (≈ 7 points)

1. Draw the graph of the function $f(x, y) = (x^2 + y^2)^{1/4}$.

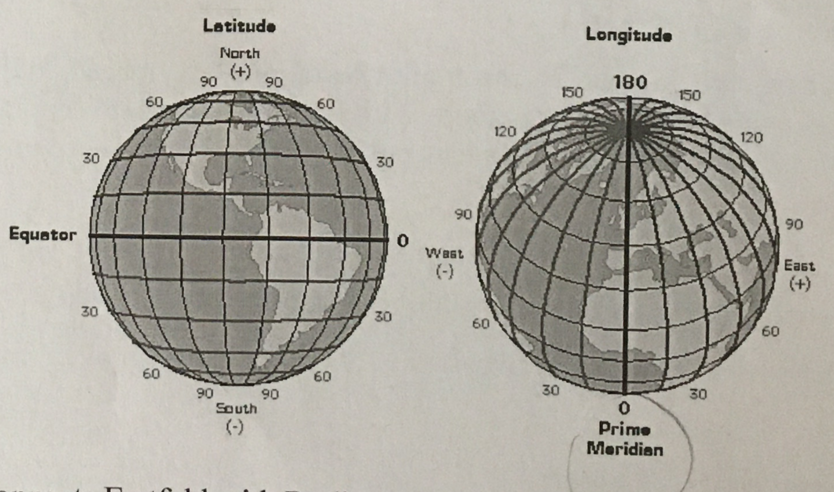
Let r, θ and z be the cylindrical coordinates and consider the following parametric curve:

$$\begin{cases} r(t) = e^{-2t} \\ \theta(t) = 2\pi t \\ z(t) = e^{-t} \end{cases} \qquad t \in \mathbb{R}^+$$

2. Express this parametric curve in cartesian coordinates, i.e. compute $x(t)$, $y(t)$ and $z(t)$.
3. Show that this parametric curve is on the graph of f.
4. Let $M(t)$ be the point of coordinates $x(t)$, $y(t)$ and $z(t)$. Write the vector $\overrightarrow{OM}(t)$ in cartesian coordinates and in the cartesian frame.
5. What is the velocity $\vec{v}$ of the point $M(t)$?
6. Give the expression of the vectors $\vec{e_r}(t)$, $\vec{e_\theta}(t)$ and $\vec{e_z}(t)$ of the local cylindrical frame at the point $M(t)$.
7. Write the vector $\overrightarrow{OM}(t)$ in cylindrical coordinates in the cylindrical frame.
8. Deduce from it the velocity $\vec{v}$ of the point $M(t)$ in cylindrical coordinates and in the cylindrical frame.
9. Give an approximate representation of the trajectory of $M(t)$.

Exercise 5 (≈ 4 points)

In geography, latitude and longitude are used to describe locations on Earth's surface, as shown in the Figure. The center of the Earth being the origin of the frame, angles describe the location of a point on Earth relative to the equator (xy-place) and the prime meridian (xz-plane).



The Gotthard Base Tunnel is a railway tunnel through the Alps in Switzerland. It is the world's longest railway and deepest traffic tunnel. It connects Erstfeld with Bodio.

Place	Altitude	Latitude	Longitude
Erstfeld	1665 m	46.825270° North	8.645389° East
Bodio	321 m	46.378313° North	8.910725° East

1. Approximating the Earth to a perfect sphere of radius R equal to 6 378 km at altitude 0, give the spherical coordinates of Erstfeld and Bodio.
2. Give the cartesian coordinates of both places.
3. Deduce the length of the Gotthard Base Tunnel, in km.