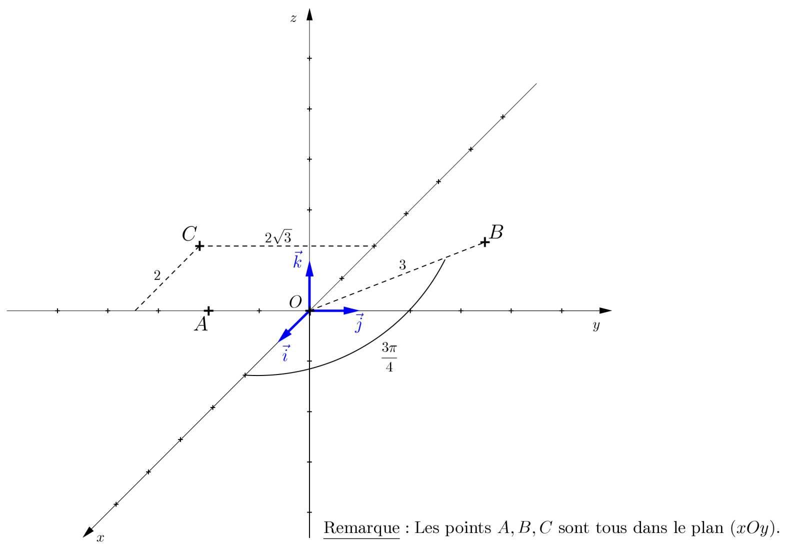
FIMI
Schoolyear 2021-2022

2nd MTES exam - SCAN FIRST
January, 14 (1 h 30 min)
Correction and Grading scale

Exercise 1 : coordinate systems [4 pts.]

1. 	0.25 (for a correct direct frame) 0.25 per point
2. $\overrightarrow{OA} = r_A \sin\theta_A \cos\varphi_A \vec{i} + r_A \sin\theta_A \sin\varphi_A \vec{j} + r_A \cos\theta_A \vec{k} = -2\vec{j}$.	0.5
so the Cartesian coordinates of A are $(0, -2, 0)$	0.25
$\overrightarrow{OB} = r_B \cos\theta_B \vec{i} + r_B \sin\theta_B \vec{j} + z_B \vec{k} = -\frac{3\sqrt{2}}{2}\vec{i} + \frac{3\sqrt{2}}{2}\vec{j}$.	0.5
So the Cartesian coordinates of B are $\left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}, 0\right)$	0.25
$r_C = \lVert\overrightarrow{OC}\rVert = \sqrt{(-2)^2 + (-2\sqrt{3})^2} = 4$ and $\begin{cases} x_C = -2 = r_C \sin\frac{\pi}{2}\cos\frac{4\pi}{3} \\ y_C = -2\sqrt{3} = r_C \sin\frac{\pi}{2}\sin\frac{4\pi}{3} \\ z_C = 0 = r_C \cos\frac{\pi}{2} \end{cases}$ so the spherical coordinates of C are $\left(r_C = 4, \theta_C = \frac{\pi}{2}, \varphi_C = \frac{4\pi}{3}\right)$.	$3 * 0.5$

Exercise 2 : Expansion of a Van der Waals gas [3 pts.]

1.a with $w = w_1\,dT + w_2\,dV$ with $w_1 = nC$ and $w_2 = \frac{n^2 a}{V^2}$ $\frac{\partial w_1}{\partial V} = 0$ and $\frac{\partial w_2}{\partial T} = 0$ so $\frac{\partial w_1}{\partial V} = \frac{\partial w_2}{\partial T}$. Therefore, w is closed.	0.75
w is closed and the domain is simply connected so w is exact.	0.5
1.b $\frac{\partial U}{\partial T} = nC$ and $\frac{\partial U}{\partial V} = \frac{n^2 a}{V^2}$. This gives $U(T,V) = nCT - \frac{n^2 a}{V} + Constant$.	1 (0.75 if no constant)
2 $q = w + P\,dV = nC\,dT + \frac{n^2 a}{V^2}dV + P\,dV = nC\,dT + \left(\frac{n^2 a}{V^2} + P\right)dV$. But $\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$, so $q = nC\,dT + \frac{nRT}{V-nb}dV$. Hence $q = nC\,dT + \frac{nRT}{V-nb}dV$	0.25
$\frac{\partial w_1}{\partial V} = 0$ and $\frac{\partial w_3}{\partial T} = \frac{nR}{V-nb} \neq 0$.So $\frac{\partial w_1}{\partial V} \neq \frac{\partial w_3}{\partial T}$, which means q is not closed.	0.5

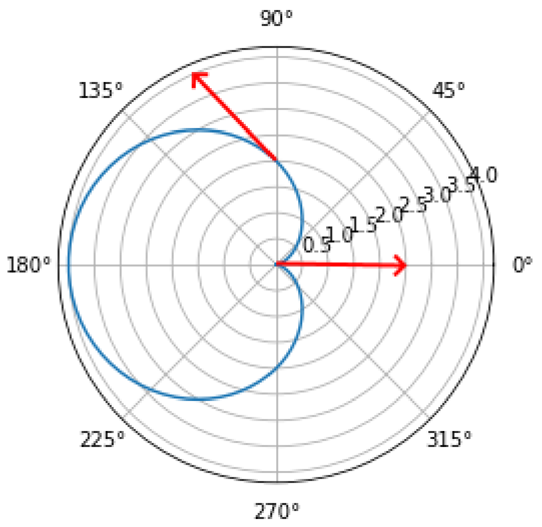
Exercise 3 : Differential calculus [6 pts.]

1. $S = 2\pi R^2 + 2\pi Rh$	0.5
2.a. differential $dA = 2\pi\big((2R+h)\,dR + R\,dh\big)$	1
uncertainty : $\Delta A = 2\pi\big((2R+h)\Delta R + R\Delta h\big)$	0.5
2.b. $\Delta A \approx 270\ \text{cm}^2$	0.5
$A = (11.6 \pm 2.7)\ \text{dm}^2$	0.5
3. cylinder volume : $V = \pi R^2 h$	0.5

differential : $dV = \pi R\big(2h\,dR + R\,dh\big)$	0.5
$A = constant : dA = 0$ implies $dh = -\frac{2R+h}{R}\,dR$ Hence $dV = \pi R\big(2h\,dR + R\,dh\big) = \pi R\Big(2h - (2R+h)\Big)dR = \pi R(h-2R)dR$	0.5 0.5
we find $dV = 0$ when $R = \frac{h}{2}$	0.5
we can check this corresponds to a maximum value of V by studying the sign of $\frac{\partial V}{\partial R}$	0.5

Exercise 4 : Cardioid [5 pts.]

1. we use Cartesian coordinates : $x(t) = r\cos(t)$ and $y(t) = r\sin(t)$. $x'(t) = 4\sin(t)\cos(t) - 2\sin(t)$ and $y'(t) = 2\sin^2(t) - 2\cos^2(t) + 2\cos(t)$	0.5
$t = \pi/2$ is a regular point : the tangent vector is $\vec{T_{\pi/2}}\big(x'(\pi/2) = -2, y'(\pi/2) = 2\big)$	0.5
$t = 0$ is a singular point as $x'(0) = 0$ and $y'(0) = 0$, so we have to compute $x''(t)$ and $y''(t)$ $x''(t) = 2\big(\cos^2(t) - \sin^2(t)\big)$ and $y''(t) = 8\cos(t)\sin(t)$	0.5
so the tangent vector is $\vec{T_0}\big(x''(0) = 2, y''(0) = 0\big)$	0.5
2. $\vec{OM} = r\vec{e_r}$	0.5
$\vec{v} = \frac{d\vec{OM}}{dt} = \frac{dr}{dt}\vec{e_r} + r\frac{d\vec{e_r}}{dt}$ with $\frac{d\vec{e_r}}{dt} = \frac{d\theta}{dt}\vec{e_\theta}$ Hence $\vec{v} = 2\sin(t)\vec{e_r} + 2(1-\cos(t))\vec{e_\theta}$	1

 3.	0.5 (curve) +0.5 (vectors)

Exercise 5 [2 pts.]

1. $\frac{\partial^2 E}{\partial z^2} = -k^2 E_0 e^{j(\omega t - kz)}$ and $\frac{\partial E}{\partial t} = j\omega E_0 e^{j(\omega t - kz)}$	1
2. $k^2 = -j\omega\mu\gamma$	1
3. $k = \pm\sqrt{\omega\mu\gamma}\,e^{\frac{-j\pi}{4}} = \pm\sqrt{\frac{\omega\mu\gamma}{2}}(1-j)$	0.5