INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

Scan First
Year 2021-2022

MTES – Exam #2

January 14, 2022 **Duration: 1 h 30**

No document allowed. No mobile phone. ***Calculator allowed****. The proposed grading scale is only indicative.*

Exercise 1 : Coordinate systems (≈ 4 points)

The points A, B, C are defined by their spherical, cylindrical and Cartesian coordinates, respectively:

$$A\left(r_A = 2;\ \theta_A = \frac{\pi}{2}; \varphi_A = \frac{3\pi}{2}\right)$$
$$B\left(r_B = 3;\ \theta_B = \frac{3\pi}{4}; z_B = 0\right)$$
$$C\left(x_C = -2;\ y_C = -2\sqrt{3}\,; z_C = 0\right)$$

1. Place the points A, B, C on a scheme and define clearly their coordinates on the scheme.
2. Give the Cartesian coordinates of points A and B, and the spherical coordinates of point C (literal expression then numerical value).

Exercise 2 : Expansion of a Van der Waals gas (≈ 3 points)

Let w and q be two differential forms defined as :

$w = nCdT + \frac{n^2a}{V^2}dV$ and $q = w + PdV$

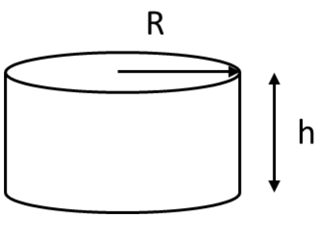
with P, V, T strictly positive quantities and n, C, a, b, R are strictily positive constants linked to each other by the relation : $\left(P + \frac{n^2a}{V^2}\right)(V - nb) = nRT$

1. a. Show that w is an exact form, that can be written dU on the domain $\Delta = \{(T, V) \in \mathbb{R}^2 \,/\, T > 0; V > nb\}$.
   b. Determine the expression of the functions $U(T, V)$

2. a. Write q as a function of T, V, dT and dV.
   b. Is q an exact form on Δ ?

Exercise 3 : Differential calculus (≈ 6 points)

Let consider a cylinder of radius R and height h, **closed** at both ends.



1. Express the total area A of the cylinder (take into account the lateral area and the areas of the discs at both ends).

2. R and h are measured with the respective uncertainties ΔR and Δh.
   a. Give the expression of the absolute uncertainty ΔA on the area.
   b. Numerical application: $R = (8.0 \pm 1.0)\ cm$ and $h = 15\ cm$ is measured with a relative uncertainty of 10%. Write the area as $A = (\ldots \pm \cdots)\ unit$.

3. A manufacturer of cans wants to obtain a maximum volume for a given area. Which relation must then verify R and h? You can express dh as a function of dR by writing that the area is constant (given area), then study the sign of dV.

Exercise 4: cardioid (≈ 4 points)

A cardioid is a plane curve traced by a point on the perimeter of a circle that is rolling around a fixed circle of the same radius. It can be described using the following equations in polar coordinates:

$$\begin{cases} r(t) = 2(1 - cost) \\ \theta(t) = t \end{cases} \quad \text{with } t \in \mathbb{R}$$

1. Give the coordinates of the vectors that are tangent to the curve for $t = 0$ and $t = \frac{\pi}{2}$.

2. With M a point on the cardioid, and O the center of the Cartesian frame, express $\overrightarrow{OM}(t)$ and $\vec{v}(t)$ in the polar local frame.

3. Graph the curve for $t \in [0; 2\pi[$. On the graph, place also the vectors computed in question 1.

Exercise 5 (≈ 3 points)

In physics, the following equation is used to describe the propagation of an electric field E in a conducting material:

$$\frac{\partial^2 E}{\partial z^2} = \mu\gamma \frac{\partial E}{\partial t}$$

To solve this equation, we use the complex expression $E = E_0 e^{j(\omega t - kz)}$ with $j^2 = -1$, t is the time, z the position in the conductor, $E_0 \neq 0$ the amplitude of the electric field for $t = 0$ and $z = 0$. γ, μ and ω are strictly positive real constants whereas k is a complex constant.

1. Express $\frac{\partial^2 E}{\partial z^2}$ and $\frac{\partial E}{\partial t}$ in function of E_0, ω, t, k and z.

2. Deduce the equation verified by k.

3. Express k as $k = k' + jk''$ with k' and k'' real numbers.