1st MTES exam - SCAN FIRST
November 26, 2021 (1 h)
Correction and Grading scale

Exercise 1 [3 pts.]

1. $u = \sqrt{2}e^{\frac{i\pi}{4}}$ and $v = 2e^{\frac{2i\pi}{3}}$	$0.5 + 0.5$
2. $\lvert\frac{u}{v}\rvert = \frac{1}{\sqrt{2}}$ and $Arg\,\frac{u}{v} = \frac{-5\pi}{12}$	$0.5 + 0.5$
3. $\frac{u}{v} = \frac{-1+\sqrt{3}}{4} - \frac{1+\sqrt{3}}{4}$ by identification, $\cos\frac{-5\pi}{12} = \frac{-1+\sqrt{3}}{2\sqrt{2}}$ and $\sin\frac{-5\pi}{12} = -\frac{1+\sqrt{3}}{2\sqrt{2}}$	0.5 $0.25 + 0.25$

Exercise 2 [2 pts.]

1. $z_0^4 = \overline{z_0}$ and $z_0^3 = \overline{z_0^2}$	$0.25 + 0.25$
2. Points A_1 to A_4 belong to the circle of center O and radius 1 As $\lvert z_0\rvert = \lvert z_0^2\rvert = \lvert z_0^3\rvert = \lvert z_0^4\rvert = 1$ 3. figure with points A_0 to A_4 Points A_1 and A_4 are symmetric wrt the x-axis, as well as points A_2 and A_3	0.5 0.5 0.5 graded on the scheme

Exercise 3 [5 pts.]

1. $\Delta = 3 - 4i$ with $\delta = \alpha - i\beta, \begin{cases} \alpha^2 - \beta^2 = 3 \\ \alpha^2 + \beta^2 = 5 \\ 2\alpha\beta = -4 \end{cases}$ This gives $\alpha = 2$ and $\beta = -1$ The solutions are $z_1 = 1 - i$ and $z_2 = 3 - 2i$	0.5 0.5×3 0.5 $0.5 + 0.5$
2. $z_1 = \sqrt{2}\exp\frac{-i\pi}{4}$ $\lvert z_2\rvert = \sqrt{13}$ $Arg\,z_2 = \arctan\frac{-2}{3} \approx -33.7° \approx -0.588$ rad	$0.25 + 0.25$ 0.5 0.5 $BONUS$ 0.5 (justification $-\frac{\pi}{2} < Arg(z_2) < \frac{\pi}{2}$)

Exercise 4 [3 pts.]

If G is the centroid of ABCDS, by associativity, it is the barycenter of I(2), J(2), S(1) with I the barycenter of A(1), B(1) and J the barycenter of C(1), D(1)	0.5 (associativity)
Hence G is the barycenter of H(4), S(1)	0.5
Therefore, $HG = \frac{1}{5}HS$	0.5
$G\begin{pmatrix}115.25m\\115.25m\\27.4m\end{pmatrix}$	0.5×3

Exercise 5 [8 pts.]

1.a Scheme with the position of all the points	0.5 (ABCDEFG) +0.5 (KLM)
1.b $\vec{DM} \times \vec{DL} = \begin{pmatrix}a\\0\\-1\end{pmatrix} \times \begin{pmatrix}0\\a\\-1\end{pmatrix} = \begin{pmatrix}a\\a\\a^2\end{pmatrix}$	0.5 (formula) +0.5 (expression)
1.c Area of DLM = $\frac{1}{2}\|\|\vec{DM} \times \vec{DL}\|\| = \frac{a}{2}\sqrt{2+a^2}$	0.5 (formula) +0.5 (expression)
1.d $\vec{DM} \times \vec{DL} \perp (DLM)$	0.25
$\vec{OK} = \vec{OB} + \vec{BK} = \begin{pmatrix}1\\1\\a\end{pmatrix}$	0.5
$\vec{OK}$ and $\vec{DM} \times \vec{DL}$ are collinear, therefore $\vec{OK} \perp (DLM)$	0.25
2.a $\vec{OM} \cdot \vec{OK} = (\vec{OH} + \vec{HM}) \cdot \vec{OK} = \vec{OH} \cdot \vec{OK}$	0.5
Since $\vec{HM} \in (DLM)$ and $\vec{OK} \perp (DLM)$	0.5
2.b $\vec{OM} \cdot \vec{OK} = a$	0.5
so $\vec{OH} \cdot \vec{OK} = a = \lambda\|\|\vec{OK}\|\|^2 = \lambda(2+a^2)$ i.e. $\lambda = \frac{a}{2+a^2}$	0.5
2.c $\vec{OH} = \lambda\vec{OK} = \frac{a}{2+a^2}\begin{pmatrix}1\\1\\a\end{pmatrix}$	0.5
2.d $\vec{HK} = \vec{OK} - \vec{OH} = \frac{2+a^2-a}{2+a^2}\begin{pmatrix}1\\1\\a\end{pmatrix}$	$0.5 + 0.5$
Hence $HK = \|\|\vec{HK}\|\| = \frac{2-a+a^2}{\sqrt{2+a^2}}$	0.5