Scan First
Year 2021-2022

MTES – Exam #1

November 26, 2021 **Duration: 1 h**

No document allowed. No mobile phone. ***No calculator allowed****. The proposed grading scale is only indicative.*

Exercise 1 (≈ 3 points)

Let consider $u = 1 + i$ and $v = -1 + i\sqrt{3}$.

1. Express u and v in the exponential form.
2. Give the modulus and an argument of $^{u}/_{v}$.
3. Deduce the values of $cos\left(\frac{-5\pi}{12}\right)$ and $sin\left(\frac{-5\pi}{12}\right)$.

Exercise 2 (≈ 2 points)

Let $z_0 = e^{\frac{2i\pi}{5}}$.

A_0, A_1, A_2, A_3 and A_4 are the image-points of the following complex numbers: $1, z_0,\ {z_0}^2, {z_0}^3,\ {z_0}^4$.

1. Identify the conjugate numbers among $z_0,\ {z_0}^2, {z_0}^3,\ {z_0}^4$. Explain your answer.
2. To which circle C do points A_0, A_1, A_2, A_3 and A_4 belong? Explain your answer.
3. Make a clear scheme indicating the positions of points A_1, A_2, A_3 and A_4 on the circle. Explain which ones are symmetrically placed with respect to the (Ox) axis.

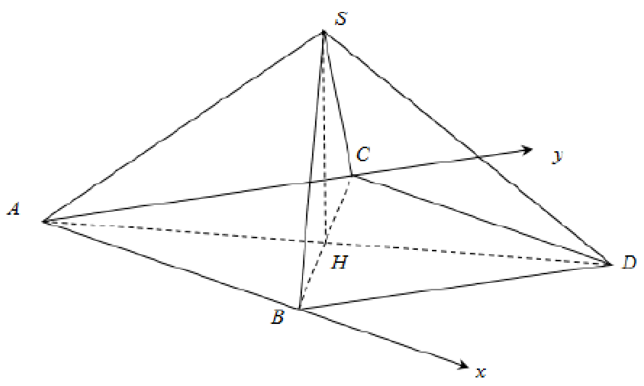
Exercise 3 (≈ 5 points)

1. Solve the equation $z^2 - (4 - 3i)z + 1 - 5i = 0$
2. Give the modulus and an argument of each solution of the equation.

Exercise 4 (≈ 3 points)

The pyramid of Cheops has a square base of 230.5 m per side and a height of 137 m. The top S of the pyramid is vertically above the center H of the square base ABDC.

Using the associativity of the barycenters, give the coordinates of the centroid of the pyramid, in the orthonormal frame of center A, represented in the figure (**explain the different steps of your demonstration**).



Exercise 5 (≈ 7 points)

Consider the space $\mathbb{R}^3$ with the orthonormal direct frame $\left(O;\ \overrightarrow{u_x} = \overrightarrow{OA};\ \overrightarrow{u_y} = \overrightarrow{OC};\ \overrightarrow{u_z} = \overrightarrow{OD}\right)$, and the points $B(1;1;0)$, $E(1;0;1)$, $F(1;1;1)$ and $G(0;1;1)$.

a being a dimensionless positive real, the points L, M and K are defined by: $\overrightarrow{OL} = a\overrightarrow{OC}$, $\overrightarrow{OM} = a\overrightarrow{OA}$ and $\overrightarrow{BK} = a\overrightarrow{BF}$.

1.
   a. Place the points on a scheme.
   b. Compute the coordinates of $\overrightarrow{DM} \times \overrightarrow{DL}$.
   c. Deduce the area of the triangle DLM.
   d. Show that the straight line (OK) is perpendicular to the plane (DLM).

2. Let H be the orthogonal projection of O onto the plane (DLM).
   a. Prove that $\overrightarrow{OM}.\overrightarrow{OK} = \overrightarrow{OH}.\overrightarrow{OK}$
   b. The vectors $\overrightarrow{OH}$ and $\overrightarrow{OK}$ being collinear, let define λ the real such that $\overrightarrow{OH} = \lambda\overrightarrow{OK}$. Show that $\lambda = \frac{a}{2+a^2}$.
   c. Give the cartesian coordinates of H.
   d. Express $\overrightarrow{HK}$ as a function of $\overrightarrow{OK}$. Show that $HK = \frac{2-a+a^2}{\sqrt{2+a^2}}$.