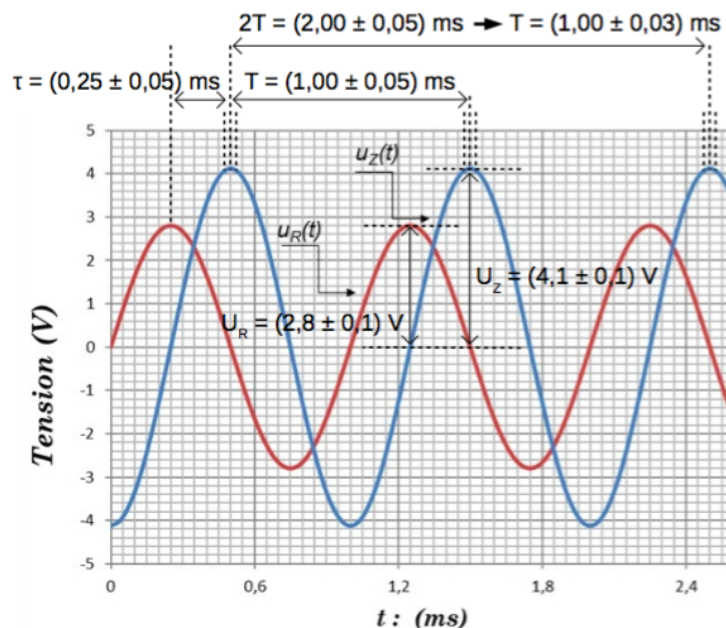


Exam 3 - Physics

December, 21st 2018, duration : 1h30

Exercise 1 : Straightforward application of lectures and practicals (5.5 points)

- | | | |
|-----|---|---------------------|
| 1.a | Taking half a graduation for left and right uncertainties (0.025 ms) we read : $T = 1.00 \pm 0.05$ ms
<i>Bonus</i> : $2T = 2.00 \pm 0.05$ ms so $T = 1.00 \pm 0.03$ ms
Time shift : $\tau = 0.25 \pm 0.05$ ms | 0.5
+0.25
0.5 |
| 1.b | $u_R = 2.8 \pm 0.1$ V and $u_Z = 4.1 \pm 0.1$ V (no score if confusion between peak and peak-to-peak amplitude)
<i>Bonus</i> : if peak-to-peak amplitude is computed to reduce uncertainties | 0.5
+0.25 |
| 2. | $u_R(t)$ and therefore $i(t)$ is ahead of $u_Z(t)$ | 0.5 |
| 3. | $\varphi = 2\pi \frac{\tau}{T}$ (or $\varphi = 360 \frac{\tau}{T}$ in degree) | 0.5 |
| 4. | $\varphi \simeq \frac{\pi}{2} \simeq 1.57$ rad (or $90^\circ \dots$)
Using the min/max method, we have $\varphi = 1.57 \pm 0.40$ rad or $\varphi = 90 \pm 23^\circ$ | 0.5
0.5 |
| 5. | Using the modulus of $\underline{u_Z}$, $\underline{u_R}$, \underline{Z} and $\underline{i(t)}$: $U_Z = ZI$ and $I = \frac{U_R}{R}$ so that : $Z = R \frac{U_Z}{U_R}$ | 0.5 |
| 6. | $Z = 10^4 \times \frac{4.1}{2.8} = 14643 \Omega$ | 0.5 |
| 7. | Given $i(t)$ is ahead of $u_Z(t)$ with a phase shift of $\pi/2$, the unknown dipole is a capacitor | 1.0 |



Exercise 2 : Nonlinear characteristic and operating point (5 points)

1. Using successive source transformation or Thevenin's theorem we have :

$$E = \frac{R_2}{R_1+R_2}E_1 - E_2 \text{ and } R = \frac{R_1R_2}{R_1+R_2}$$

1.5

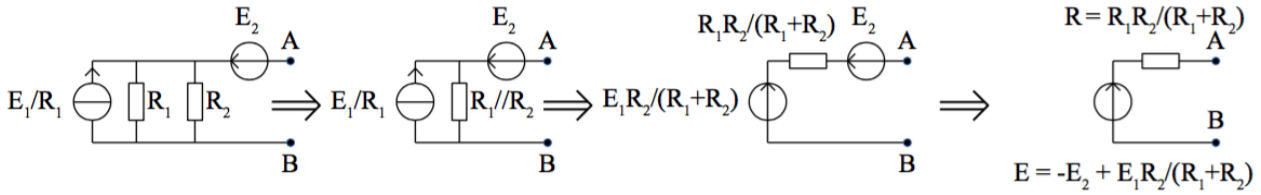


FIGURE 1 – Using successive source transformation

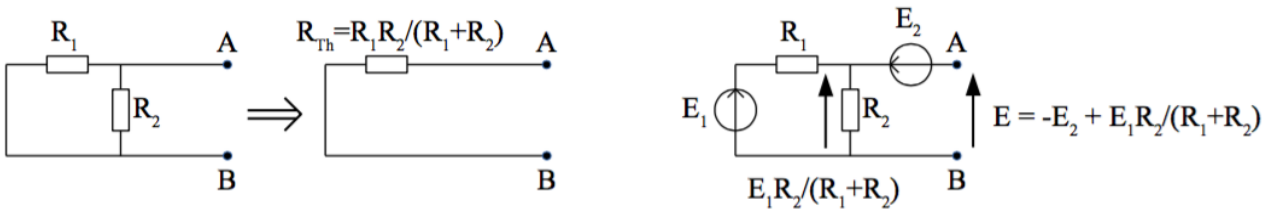


FIGURE 2 – Using Thevenin's theorem

- 2.a The orientation of I being provided, Kirchhoff circuit's law writes : $E - (R + R_L)I = V_D$
 It is supposed that $E > 0$. We can use logic and absurdity to demonstrate that $V_D > 0$.
 Assuming that $V_D < 0$ then $I \leq 0$ given the IV characteristics of the diode. Given the previous expression of V_D we have $V_D > 0$ which is absurd. The only possibility is $V_D > 0$ and $I > 0$

0.5

0.5

- 2.b Given $E - (R + R_L)I = V_D$ we have another expression of I as function of V_D :

$$I = \frac{E - V_D}{R + R_L}$$

1.0

This provide the equation of a straight line crossing the horizontal axis at $V_D = E = 1.5 \text{ V}$ and the vertical axis at $I = \frac{E}{R+R_L} = 0.05 \text{ A}$

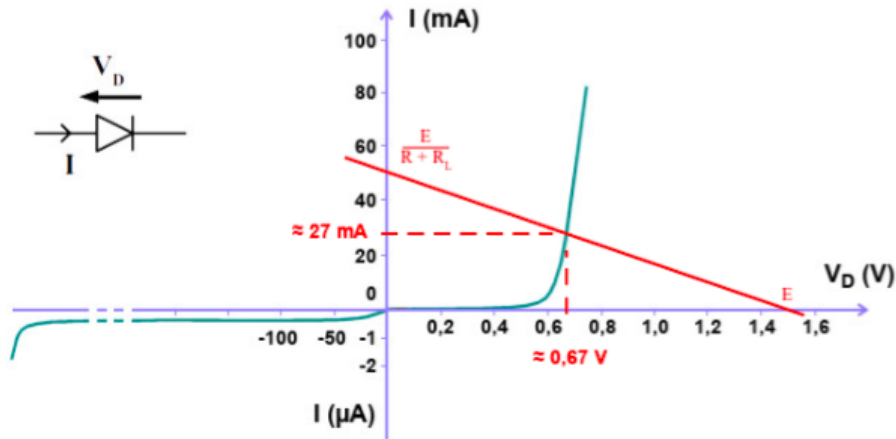
0.5

We can then determine graphically the operating point of the circuit : $V = 0.67 \pm 0.02 \text{ V}$ and $I = 27 \pm 2 \text{ mA}$

1.0

Bonus : uncertainites

+0.5



Exercise 3 : Circuit in transient regime (9.5 points)

Notice : signs of the following expression have to be adapted according to the choice of positive orientation.

1. In steady state regime : $u_{DB} = L \frac{di_{DB}}{dt} = 0 \text{ V}$, so that $u_{CB} = 0 \text{ V}$ 0.5
 $u_{AC} = u_{AD} = E = 24 \text{ V}$ 0.5
 $i_{AC} = u_{AC}/R_1 = 2 \text{ A}$ and $i_{AD} = u_{AD}/R_3 = 3 \text{ A}$ 0.5
 $i_{CB} = u_{CB}/R_2 = 0 \text{ A}$ 0.5
 $i_{CD} = i_{AC} - i_{CB} = 2 \text{ A}$ and $i_{DB} = i_{CD} + i_{AD} = 5 \text{ A}$ 0.5
 Total current supplied by E : $i_{BA} = i_{AC} + i_{AD} = 5 \text{ A}$ 0.5

2. Illustration of a possible solution : 0.5
 Once K is open we have $u_{DB} = L \frac{di_{DB}}{dt}$ 0.5
 In branch ADB : $L \frac{di_{DB}}{dt} + R_3 i_{DB} = E$ 1.0
 Computing the time derivative of the previous equation gives : $L \frac{d^2 i_{DB}}{dt^2} + R_3 \frac{di_{DB}}{dt} = 0$ (1) 1.0
 $u_{CB} = \frac{R_2}{R_1+R_2} E$ (voltage divider or Kirchoff circuit's law) 0.5
 And : $u_{CD} = u_{CB} - u_{DB} = \frac{R_2}{R_1+R_2} E - L \frac{di_{DB}}{dt}$ (2) 0.5
 We deduce : $\frac{di_{DB}}{dt} = \frac{R_2}{(R_1+R_2)L} E - \frac{1}{L} u_{CD}$ (3) and $\frac{d^2 i_{DB}}{dt^2} = -\frac{1}{L} \frac{du_{CD}}{dt}$ (4) 0.5
 Combining (1), (3) and (4) we get : $\frac{du_{CD}}{dt} + \frac{R_3}{L} u_{CD} = \frac{R_2 R_3}{(R_1+R_2)L} E$ 0.5

3. From the previous differential equation : 0.5
 $\lim_{t \rightarrow +\infty} u_{CD}(t) = \frac{R_2}{R_1+R_2} E = 8 \text{ V}$ (or $\lim_{t \rightarrow +\infty} u_{DB}(t) = 0 \text{ V}$ so that $u_{CD} = u_{CB}$)

4. Using $\tau = L/R_3$, solution : $u_{CD} = A \exp(-t/\tau) + \frac{R_2}{R_1+R_2} E$ 1.0
 Warning! : At $t = 0^+$ $u_{CD}(0^+) \neq 0 \text{ V}$ but $u_{CD}(0^+) = 24 \text{ V}$
 Indeed, u_{AC} abruptly changes from $E = 24 \text{ V}$ to $u_{AC}(0^+) = \frac{R_1}{R_1+R_2} E = 16 \text{ V}$
 The current flowing through the coil is always continuous. At $t = 0^-$ this current is equal to $\frac{E}{R_1} + \frac{E}{R_3}$. At $t = 0^+$ this current flows through resistor R_3 so that $u_{AD}(0^+) = \frac{R_1+R_3}{R_1} E = 40 \text{ V}$. Given +0.5
 $u_{CD} = u_{AD} - u_{AC}$ we have : $u_{CD}(0^+) = \left(\frac{R_1+R_3}{R_1} - \frac{R_1}{R_1+R_2} \right) E = \left(\frac{R_1 R_3 + R_1 R_2 + R_2 R_3}{R_1(R_1+R_2)} \right) E = 24 \text{ V}$
 Given this result we find $A = \frac{R_3}{R_1} E = 16 \text{ V}$ 0.5
 The plot shall exhibit an exponential decay from 24 V at $t = 0$ to 8 V . 0.5

$$u_{CD} = f(t)$$

