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CAREL Timotheé

08/03/2019

SCAN 1st Gr. 63.

1st Mechanics Test.

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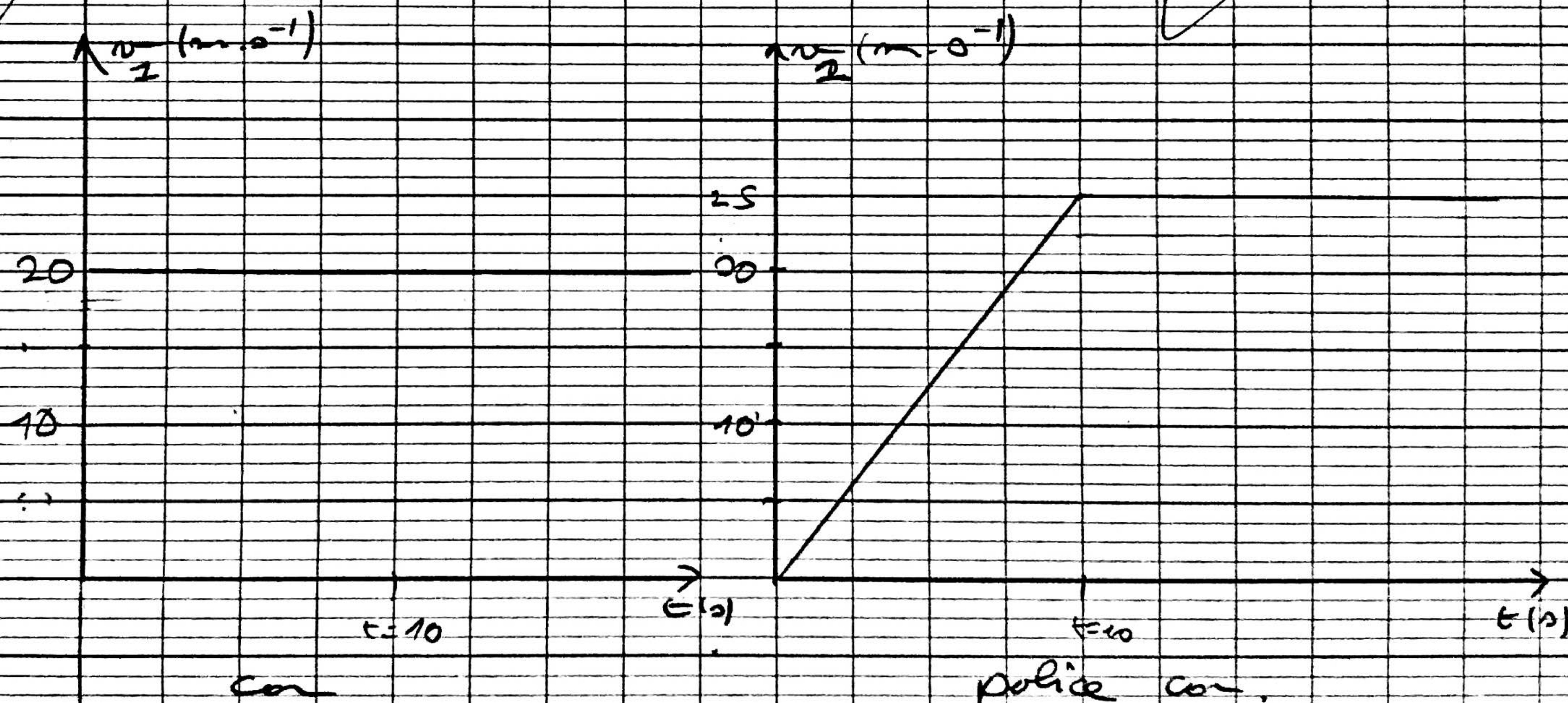
Exercice 1:



a) The police car passes the car when their position are equal. I.e we need to find a time t such that $\int_0^t v_1\,dt = \int_0^t v_2\,dt$ where v_1 and v_2 are the speeds of the police car and the car respectively.

We will call x the distance travelled since the car passed the police car and t the time since the same event.

$\int_0^t v_1\,dt = [v_1 t]_0^t = [20t]_0^t = 20t.$

Valid if $\boxed{t > 10}$.

$$\int_0^t v_2\,dt = \int_0^{10} v_2(t)\,dt + \int_{10}^{t} v_2(t)\,dt$$

$$= \underbrace{\frac{10 \times 25}{2}}_{\text{area with lines.}} + [25t]_{10}^{t} = 125 + 25t - 25 \times 10 = 25t - 125.$$

$$\int_0^t v_1\,dt = \int_0^t v_2\,dt \iff 20t = 25t - 125$$
$$\iff 5t = 125$$
$$\iff t = \frac{125}{5} = 25 \qquad (t > 10)$$

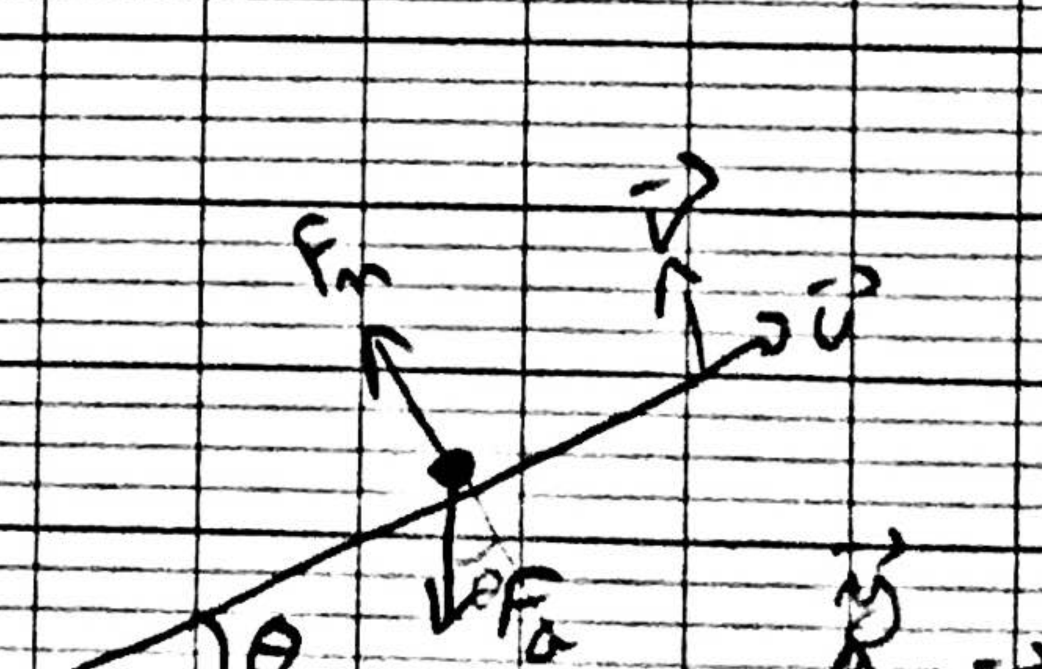
It takes 25 s. for the police car to catch up.

b) We can calculate the distance traveled by the 1st car in 25 s. by computing:

$$\int_0^{25} v_1\,dt = 20 \times 25 = 500\ m$$

They catch up 500 m after their first crossing.

2) Exercise 2:



Forces:

$\vec{F_N} = F_n \vec{V}$

$\vec{F_g} = -mg\vec{y}$

1) $\sum \vec{F} = m\vec{a}$

$\Rightarrow -mg\vec{y} + F_n\vec{v} = m\vec{a}$

$\vec{a}$ is along $\vec{u}$.

Projection on $\vec{u}$: $\vec{a} = a\vec{u}$

$-mg\sin\theta = ma \iff -g\sin\theta = a.$

2) ball at S_{max} $\Leftrightarrow$ $v = 0$.

$a = -g\sin\theta$

$v = -g\sin\theta t + v_0$.

$S = -\frac{1}{2} g \sin\theta t^2 + v_0 t + \cancel{s_0}$ 0.

We take $s = 0$ at $t = 0$.

We solve $-g\sin\theta t + v_0 = 0$ for t.

$\Leftrightarrow v_0 = g\sin\theta t$.

$\Leftrightarrow t = \frac{v_0}{g\sin\theta}$.

$S_{max} = -\frac{1}{2} g \sin\theta \frac{v_0^2}{g^2\sin^2\theta} + v_0 \frac{v_0}{g\sin\theta}$

$= -\frac{v_0^2}{2g\sin\theta} + \frac{2v_0^2}{2g\sin\theta}$

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$= \boxed{\frac{v_0^2}{2g\sin\theta}}$

3) Ball has returned to the girl's hand:

$S = 0$ and $t \neq 0$. $(t > 0)$

$S = 0 \Leftrightarrow -\frac{1}{2} g \sin\theta t^2 + v_0 t = 0$

$\Leftrightarrow t(-\frac{1}{2} g \sin\theta t + v_0) = 0$

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$\Leftrightarrow t = 0$ or $-\frac{g}{2}\sin\theta t + v_0 = 0$.

not the right root.

$\Leftrightarrow v_0 = \frac{g}{2}\sin\theta t$

$\Leftrightarrow t = \frac{v_0}{\frac{g}{2}\sin\theta} = \boxed{2\frac{v_0}{g\sin\theta}}$

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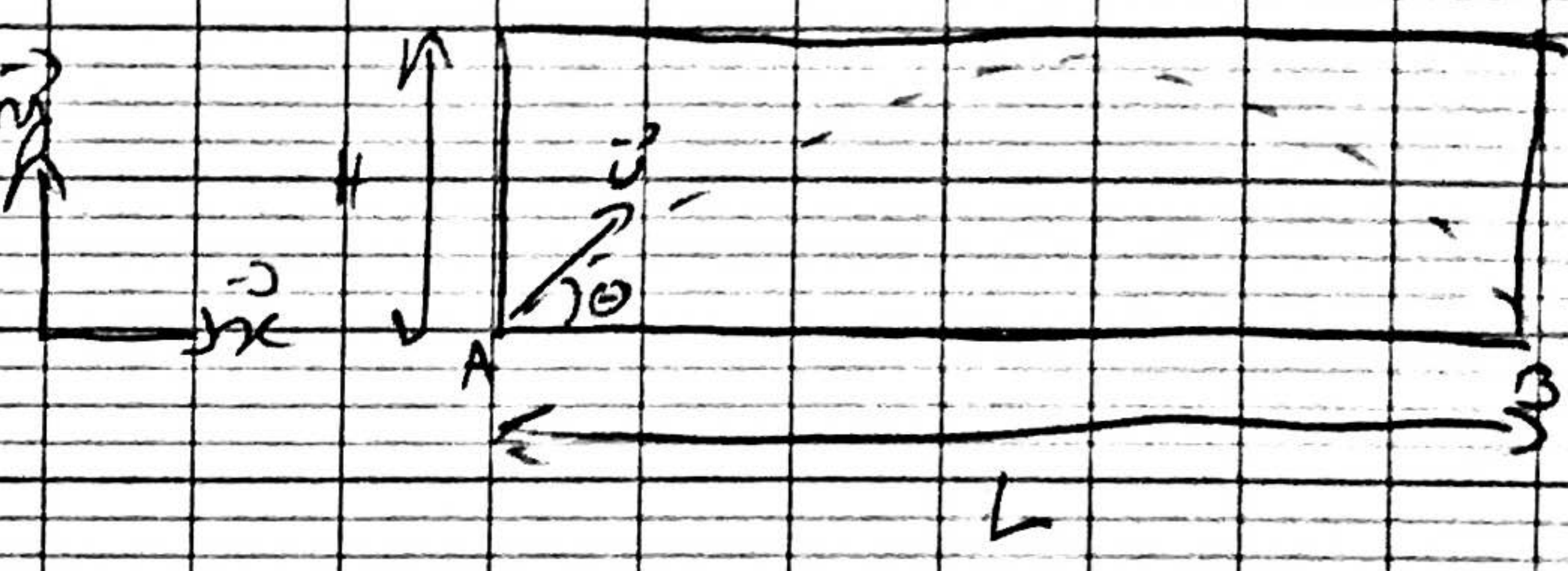
Exercise 3:

During the whole trajectory, the projectile is in free fall.
I.e. the only force exerted on it is the force exerted by gravity.

Let $\vec{x}$ be a unit vector in the horizontal direction, and $\vec{y}$ one in the vertical direction. (towards the right and up)

$\vec{F_g} = -mg\vec{y}$



$\sum \vec{F}_{ext} = m\vec{a}$

$\Rightarrow -mg\vec{y} = m\vec{a}$

$\Rightarrow \vec{a} = -g\vec{y}$

at A, $x=0$, $y=0$

On x:

$a_x = 0$

1 $v_x = u\cos\theta$

$p_x = u\cos\theta\, t$ ✓

On y:

$a_y = -g$

$v_y = -gt + u\sin\theta$

$p_y = -\frac{g}{2}t^2 + u\sin\theta\, t$ ✓

We want at t_{impact}, $p_y = 0$ and $p_x = L$. ✓

We also need: $\forall t$, $p_y \leq H$

$$\begin{cases} p_y = 0 \\ p_x = L \end{cases} \Leftrightarrow \begin{cases} -\frac{g}{2}t^2 + u\sin\theta\, t = 0 \\ u\cos\theta\, t = L \end{cases}$$

2 $$\Leftrightarrow \begin{cases} t\left(-\frac{g}{2}t + u\sin\theta\right) = 0 \\ u\cos\theta\, t = L \end{cases} \Leftrightarrow \begin{cases} t = 0 \text{ or } t = 2\frac{u\sin\theta}{g} \\ u\cos\theta \times 2\frac{u\sin\theta}{g} = L \end{cases}$$ OK ✓

$\Rightarrow t = \frac{p_x}{u\cos\theta}$ $\quad p_y(p_x) = -\frac{g}{2}\left(\frac{p_x^2}{u^2\cos^2\theta}\right) + u\sin\theta\frac{p_x}{u\cos\theta}$

$$p_y(p_x) = \frac{-g\,p_x^2}{2u^2\cos^2\theta} + p_x\tan\theta$$

We want $p_y(L) = 0$ already use (just above)

$\Leftrightarrow \frac{-gL^2}{2u^2\cos^2\theta} + L\tan\theta = 0 \Leftrightarrow \frac{gL^2}{2u^2\cos^2\theta} = L\tan\theta$

$\Leftrightarrow gL^2 = L\tan\theta\, 2u^2\cos^2\theta \Leftrightarrow u^2 = \frac{gL^2}{2L\tan\theta\cos^2\theta} = \frac{gL}{2\tan\theta\cos^2\theta}$

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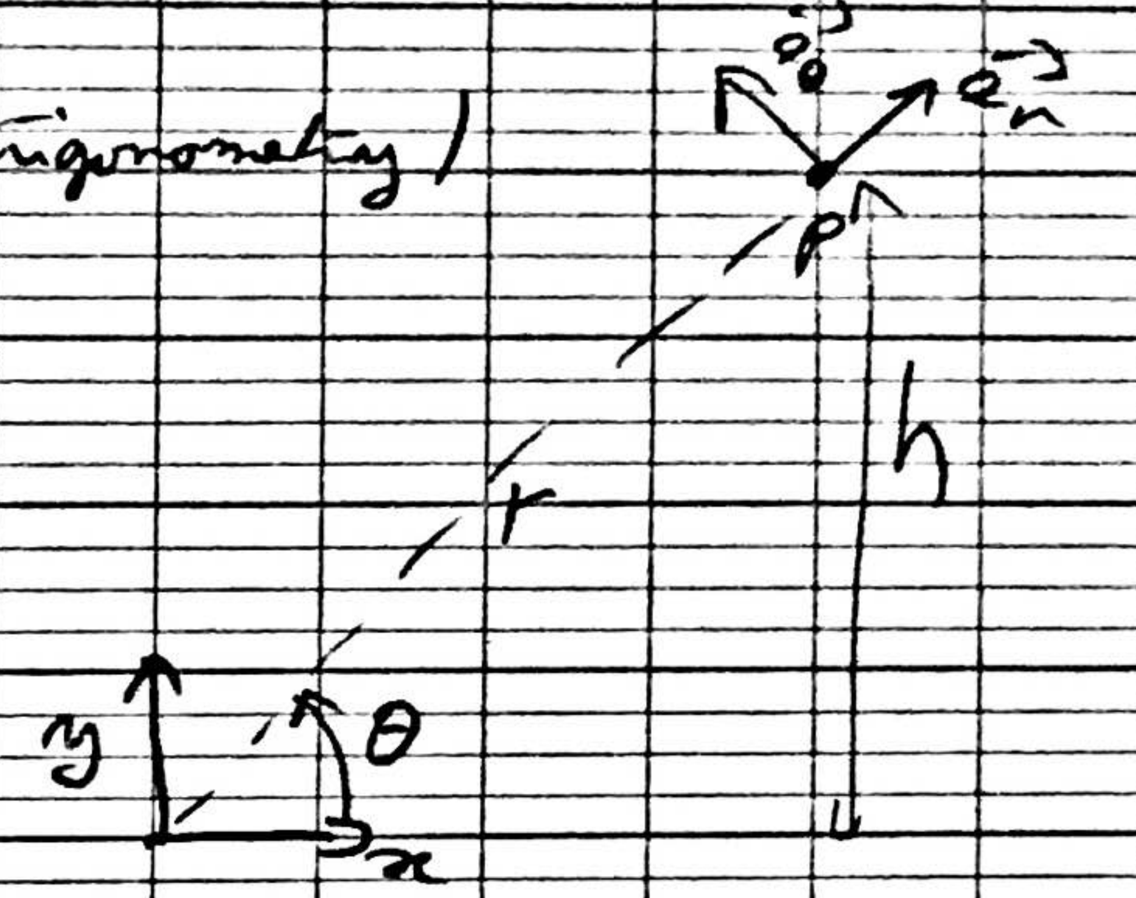
Exercice 4:

$\sin\theta = \dfrac{h}{r}$ (trigonometry)



$\Rightarrow h = r\sin\theta$

$\vec{OP} = r\vec{e_r}$

$\vec{v} = \dfrac{d\vec{OP}}{dt} = \dot{r}\vec{e_r} + r\dfrac{d\vec{e_r}}{dt} = \dot{r}\vec{e_r} + r\dot{\theta}\vec{e_\theta}$

$\cancel{\|\vec{v}\| = \sqrt{\dot{r}^2 + (r\dot{\theta})^2} = v}$