Physics - semester 1 – Exam #3

December 18th 2020 - Duration : 1h30

No document allowed. No mobile phone. ***Any type of calculator allowed****. The proposed grading scale is indicative. The marks will account not only for the results, but also for the justifications, and the way you analyze the results. Moreover, any result must be given* ***in its literal form involving only the data given in the text****. It is also reminded that the general clarity and cleanness of your paper may also be taken into account.*

Exercice 1 : Course question and direct application(≈ 4 points).

1) A doped silicon (Si) bar with a 1 mm^2 cross section is crossed by an electrical current of 4.8A. Possibly using a dimensional analysis, determine the velocity at which the charge carriers circulates in this Si bar. The charge carrier concentration is n=10^{16} electrons.cm^{-3} and the electron charge is q=$1.6 \cdot 10^{-19} C$.

2) Let be two sinusoidal voltages : $u_1 = \sqrt{3}cos(100\pi t)\ et\ u_2 = cos\left(100\pi t + \frac{\pi}{2}\right)$.
   - Give the name of the physical quantities corresponding to $\sqrt{3}, 100\pi\ and\ \frac{\pi}{2}$.
   - Are the two signals synchronous or out of phase?
   - Express $u = u_1 + u_2$ in the form $Acos(100\pi t + \varphi)$ detailing the values of A and φ.

Exercice 2 : Bipolar Transistor – common emitter configuration (≈ 7 points)

A bipolar transistor is a 3 terminals device (namely **E**=Emitter ; **B**= Base ; **C**= Collector). A typical circuit using a bipolar transistor in the "common emitter" configuration is given on Figure 1. In this exercise, we will consider an equivalent DC model of the transistor as presented in Fig. 1: the transistor can be modeled by a resistor R_B between **E** and **B** and an ideal current source $\beta \cdot i_B$ between **C** and **E** where $\beta > 0$ and i_B is the current flowing in the base (B)

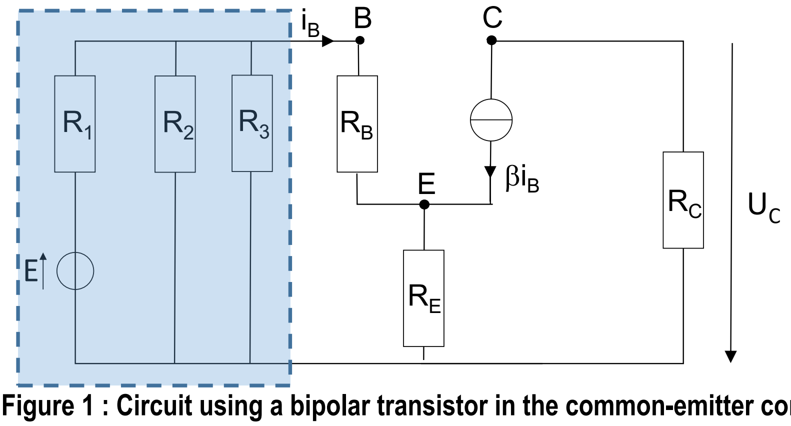


Figure 1 : Circuit using a bipolar transistor in the common-emitter configuration.
E,B and C denotes the 3 terminals of the bipolar transistor: **E**=Emitter ; **B**= Base ; **C**= Collector.

1) Replace the shaded circuit within the dashed area of Figure 1 by an equivalent Thevenin source, providing the value of its electromotive force (E_{TH}) and internal resistance (R_{TH}).

2) Considering the circuit with the equivalent Thevenin source (E_{TH} , R_{TH}), use Kirchhoff's circuit law to show that $U_C = \frac{\beta R_C}{R_B + R_{TH} + (\beta+1)R_E} E_{TH}$

3) Calculate the U_C/E_{TH} ratio together with its uncertainty using:
   R_B=R_E= R_{TH}= 1kΩ ; R_C=2kΩ. Resistance values are given with a 1% relative uncertainty.
   β=10 is supposed to have no uncertainty.

Provide the result as: U_C/E_{TH} = (....+/-....) unit.

4) What is the main interest for this circuit?

Exercice 3 : Current-voltage characteristic of a photovoltaic cell (≈ 9 points).

A photovoltaic cell or PV cell, used for assembling solar panels, converts the light energy it receives into electrical energy. Therefore, its current-voltage characteristic $\mathrm{I} = \mathrm{f(V)}$ depends on its illumination (denoted **E**). Figure 2 shows the current-voltage characteristic of L-side square-shaped PV cell with $\mathrm{L} = (10{,}0 \pm 0{,}1)\ \mathrm{cm}$ for $\mathrm{E} = 800\ \mathrm{W} \cdot \mathrm{m}^{-2}$ (which corresponds to a typical sunny day).

1) Based on the I-V curve of Figure 2, how can we qualify a photovoltaic cell: what type of dipole is it ?

2) Specify directly in Figure 2, justifying, the region(s) in which this photovoltaic cell operates as a receiver and / or as a generator.

3) Suggest a measurement protocol that would permit to obtain the characteristic given in Figure 2 to be traced.

4) The conversion efficiency η of the cell is defined by the ratio between $\boldsymbol{P_L}$ the light power received by the cell and $\boldsymbol{P_e}$, the electric power supplied by the cell: $\eta = \frac{P_L}{P_e}$.

> a). Exploiting a dimensional analysis, deduce the expression of $\boldsymbol{P_L}$ from the above defined physical quantities and give the literal expression of the conversion efficiency η as function of the physical quantities stated above.
>
> b). From the I-V curve of Figure 2, what is η worth when the solar cell is connected to a resistor R = 150 mΩ? You will carefully account for the uncertainties.

5) *Unguided Question*:
Assume that one can associate at will (in series and in parallel) photovoltaic cells to build a solar panel: in each branch the cells are connected in series, and the different branches are connected parallel.
This solar panel is intended to power a pump that will draw water to irrigate an isolated land. In order to work properly, the pump must be supplied with a DC voltage of 12 V and an electric power of 60 W.
Propose an arrangement of the photovoltaic cells allowing this pump to operate optimally.

Your answer will be argued and illustrated with electrical scheme. Even if you do not go to the end of the question, the problem-solving approach will be evaluated here.

NB : To get you started, as with any problem solving, you can:
- *Make a schematic electrical circuit of the desired installation.*
- *Think about the physical quantities useful in this problem, name them precisely and make them appear on the diagram.*
- *Think about the constraints imposed by the statement.*
- *Rephrase the question.*
- *Suggest a resolution path.*
- *Implement it.*

NAME : First name :

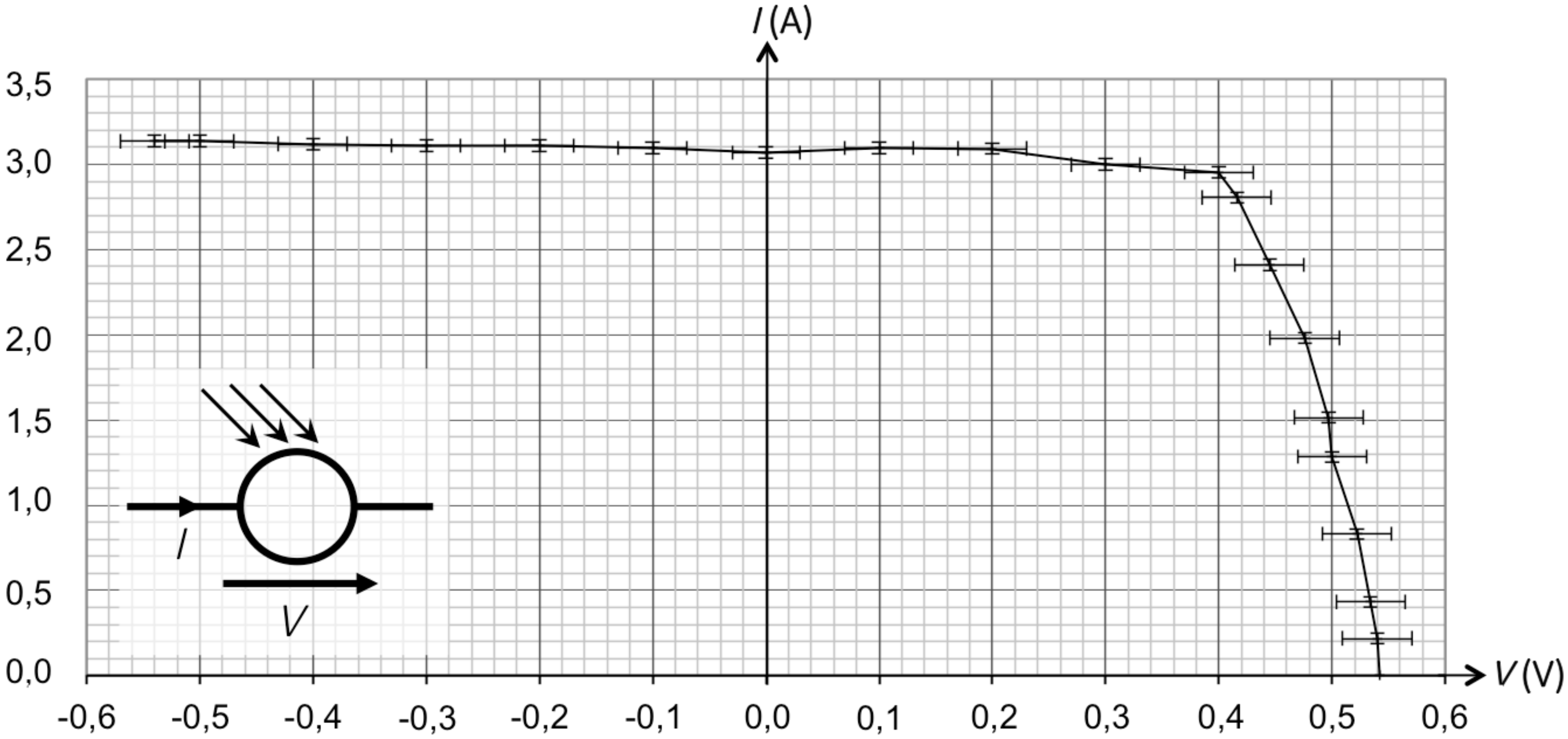


Figure 2 : Current-Voltage of a photovolatic cell subjected to an illumination E = 800 $W.m^{-2}$.

The characteristic was obtained using the convention indicated on the scheme

Correction

Exercice 1 : Course question and direct application — 4 points

1) $I = qnvS$ where $v = \frac{I}{qnS}$ ………………………………..... **0.5pts**

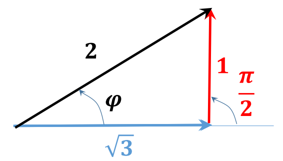
$v = \frac{4.8}{10^{16}*1.6*10^{-19}*10^{6}}$ =3000 m.s^{-1} …………………….. **0.5pts**

2)

2.1 $\sqrt{3}$ *is an* amplitude, 100π is an angular frequency and $\frac{\pi}{2}$ is a phase-shift — **1.0pts** (0.5 for 2/3, 0 for 1/3 of correct answers)

2.2 u_1 and u_2 are out of phase $(\frac{\pi}{2})$ ……………. **0.5pts**

2.3 Choosing either phasor diagram or complex calculations : A=2 ; $\varphi = \frac{\pi}{6}$ ……... — **0.5pts for A** **0.5pts for φ**



$$\cos(\varphi) = \frac{\sqrt{3}}{2}$$
$$\text{sins}(\varphi) = \frac{1}{2}$$

0.5pts scheme

Or

0.5pts complex calculation

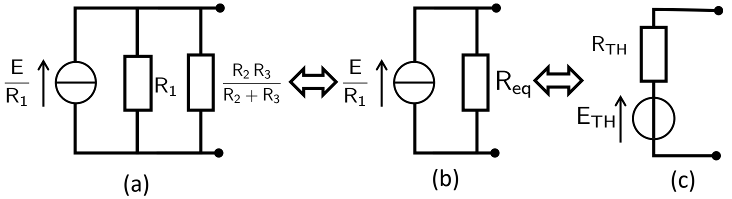
Or

$$\sqrt{3}e^{j100\pi t} + e^{j(100\pi t+\frac{\pi}{2})} = (\sqrt{3}+j)e^{j100\pi t} = 2e^{j\frac{\pi}{6}}e^{j100\pi t}$$

Exercice 2 : Bipolar Transistor – common emitter configuration — 7 points

<u>**Q1: 2.0 pts**</u>

1) Using the successive transformations depicted below :



(1.0 for E_{TH} 1.0 for R_{TH})

(a) combining R_2 and R_3 in parallel and transforming the Thevenin into a Norton Source

(b) Combining the resistances in parallel: $R_{eq} = \frac{R_1R_2R_3}{R_1R_2+R_2R_3+R_1R_3}$

(c) Moving back to a Thevenin source:

$E_{TH} = \frac{R_2R_3}{R_1R_2+R_2R_3+R_1R_3}E$ and $R_{TH} = R_{eq} = \frac{R_1R_2R_3}{R_1R_2+R_2R_3+R_1R_3}$

2) There are 2 loops in the circuit (see scheme below). From Ohm's law across Rc:

Q2: 2.5 pts

$$U_C = R_C \cdot \beta \cdot i_B \quad (1)$$

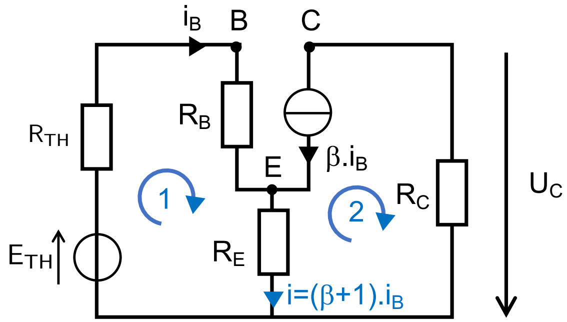
We then have to determine i_B as function of the other quantities defined in the circuit.

0.5 pts

Let I be the current flowing in R_E (see scheme below). I can be determined using Kirchhoff's current law at node E (see scheme below):

$$I = i_B + \beta \cdot i_B = (\beta + 1) \cdot i_B$$

1.0 pts
(0.5pts defining I + 0.5pts finding its value)



Then using Kirchhoff's voltage law in loop 1:

$$E_{TH} - (R_{TH} + R_B) \cdot i_B - (\beta + 1) \cdot R_E \cdot i_B = 0$$

Therefore :

0.5 pts

$$i_B = \frac{E_{TH}}{R_B + R_{TH} + (\beta+1)\cdot R_E} \quad (2)$$

Combining (1) and (2) gives :

0.5 pts

$$U_C = \frac{\beta \cdot R_C}{R_B + R_{TH} + (\beta + 1) \cdot R_E} E_{TH}$$

3) Given the uncertainties on resistances:

Q3: 2.0 pts

$$\left(U_C/E_{TH}\right)_{min} = \frac{\beta \cdot R_{C-min}}{R_{B-max} + R_{TH-max} + (\beta + 1) \cdot R_{E-max}} \approx 1.50800 \ldots$$

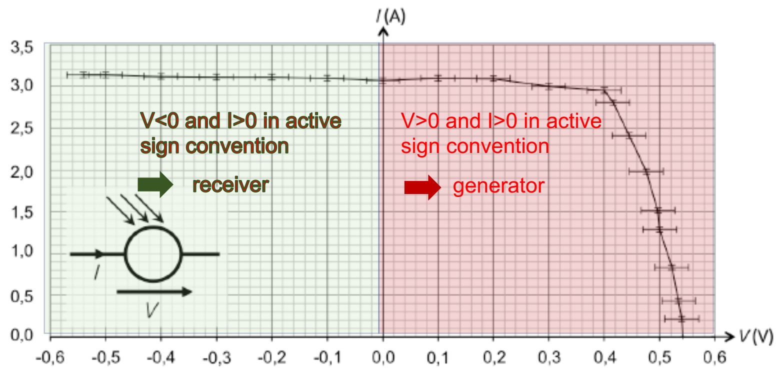
0.5 pts
(expression+value)

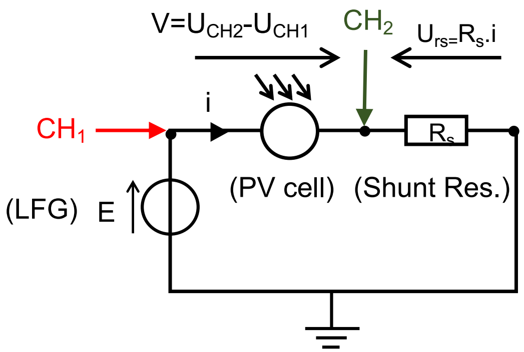
$$\left(U_C/E_{TH}\right)_{max} = \frac{\beta \cdot R_{C-max}}{R_{B-min} + R_{TH-min} + (\beta + 1) \cdot R_{E-min}} \approx 1.56954 \ldots$$

0.5 pts
(expression+value)

$$\left(U_C/E_{TH}\right)_{mean} = \frac{1}{2}\left[\left(U_C/E_{TH}\right)_{max} + \left(U_C/E_{TH}\right)_{min}\right] \approx 1.53877 \ldots$$

$$\Delta\left({}^{U_C}/_{E_{TH}}\right)_{mean} = \frac{1}{2}\left[\left({}^{U_C}/_{E_{TH}}\right)_{max} - \left({}^{U_C}/_{E_{TH}}\right)_{min}\right] \approx 0.03077$$	**0.25 pts**
Conclusion : $${}^{U_C}/_{E_{TH}} = (1.539 \pm 0.031)$$ or $${}^{U_C}/_{E_{TH}} = (1.54 \pm 0.04)$$	**0.25 pts**
${}^{U_C}/_{E_{TH}}$ has no dimension nor unit since it is a voltage ratio	**0.5 pts** (result + unit)
4) Given ${}^{U_C}/_{E_{TH}} > 1$ this circuit can be used as a voltage amplifier	**<u>Q4: 0.5 pt</u>**

Exercice 3 : Current-voltage characteristic of a photovoltaic cell	**9→10 points**
1) Given the I-V curve of the dipole does not cross (0;0), this dipole is then an active one.	**<u>Q1: 0.5 pt</u>**
2) 	**<u>Q2: 1.0 pt</u>**
3) The IV curve can be obtained using a setup involving either an oscilloscope or a voltmeter. In the following, a setup using an oscilloscope is described, the latter could be replaced by a voltmeter to measure the voltage drop across the PV cell or the shunt resistor. A shown in the scheme below, the general setup requires a Low Frequency Generator (LFG), a shunt resistor (Rs) and an oscilloscope:	**<u>Q3: 2.0 pt</u>**



On CH1 the voltage across the LFG is measured while CH2 measures the voltage across the shunt resistor.
In order to construct the point-by-point IV curve of the dipole, the following protocol is proposed:

- A voltage E is set at the LFG
- The current flowing through the PV cell is obtained by measuring on CH_2 the voltage difference U_{RS} across the shunt resistor ($I = U_{RS}/ R_S$)
- The voltage difference V, in active sign convention, is accessed from CH_2-CH_1 using the MATH menu of the oscilloscope
- This procedure is repeated for several E values (positive and negative) in order to explore the full IV curve of the dipole.

4) **a.** Given E is a power by unit surface, the light power received by the PV cell writes:

$$P_L = E \cdot L^2$$

The efficiency ratio writes:

$$\eta = \frac{E \cdot L^2}{P_e}$$

Note: $P_e = V.I$ in active sign convention.

b. Given the uncertainty on L, the light power lies in the following range:

$$E \cdot L_{min}^2 \leq P_L \leq E \cdot L_{max}^2$$
$$7.84\ W \leq P_L \leq 8.16\ W$$

To determine η, the electrical power P_e delivered by the PV cell has then to be determined.
To this aim, we will find the operating point of the PV cell as it is connected to a 150 mΩ resistor.
This can be achieved using a graphical method by finding the intersection point between the IV curve of the PV cell and the IV curve of the resistor.

Instruments description+ annotated scheme:
1.0pts

1.0pts
Measurement protocol

<u>Q4a: 0.5 pts</u>
0.25pts (P_L expression)

0.25pts (η expression)

<u>Q4b: 2.5 pt</u>

0.25pts (uncertainty range of P_L)

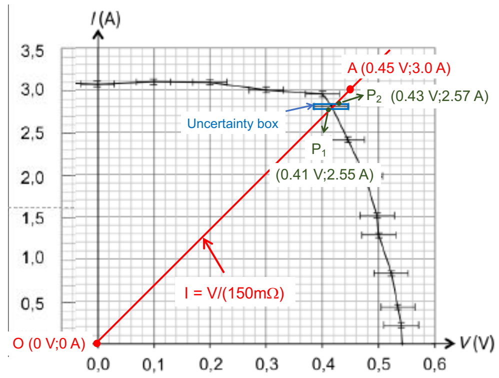
0.25pts (graphical method statement)

The IV curve of the resistor should pass through the origin (O) and point
A = (0.45 V, 0.45/.150 = 3.0 A).

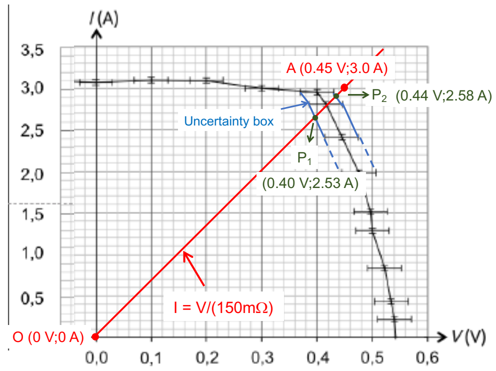
0.5pts (IV curve of 150mΩ resistor)

As depicted in the schemes above, 2 methods can be accepted to find the intersection point of both curves.

(a) Using the closest uncertainty box



(b) Joining uncertainty boxes



Method (a) exploits the closest uncertainty box. Limiting ourselves to half a graduation for both I and V values (resp. ΔV=0.1V and ΔI=0.05A), the 2 extreme operating points are:

P_1 = (0.38 V; 2.75 A) and P_2 = (0.46 V; 2.85 A)

Therefore :

$$1.0455\ W \leq P_e \leq\ 1.311\ W$$

Rounding down the lowest power and up the highest one leads to:

$$1.045\ W \leq P_e \leq\ 1.311\ W$$

0.5pts (finding possible operating points from graphical method)

Method (b) exploits the uncertainty boxes in the vicinity of the intersection point, considering that the “real” I-V curve of the cell stands within the area defined by joining the uncertainty boxes.
Through this second method we find the following extremes operating points:

P_1 = (0.40 V; 2.53 A) and P_2 = (0.44V; 2.58 A)

Therefore :

$$1.0120\ W \leq P_e \leq\ 1.1352\ W$$

Rounding down the lowest power and up the highest one leads to:

$$1.012\ W \leq P_e \leq\ 1.136\ W$$

0.5pts (uncertainty range of Pe)

Given the values found for P_e and P_L, the efficiency ratio, the conversion efficiency is enclosed within the following range:

$$\frac{P_L^{min}}{P_e^{max}} \leq \eta \leq \frac{P_L^{max}}{P_e^{min}}$$

More specifically:

From **Method (a):**

$$5.98 \leq \eta \leq 7.81$$

From **Method (b):**

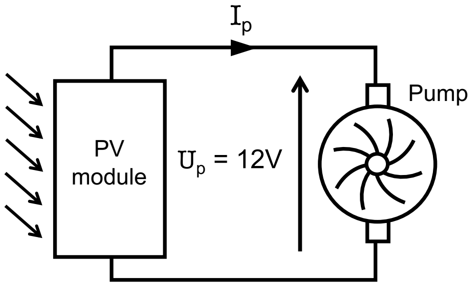
$$6.90 \leq \eta \leq 8.07$$

0.5pts (uncertainty range of η)

.

Q5: 3.5 pts

5) Let U_P and I_P be the DC voltage and the electrical current feeding the pump.
A scheme of the desired circuit is depicted below:



0.25pts (scheme + definition of I_p)

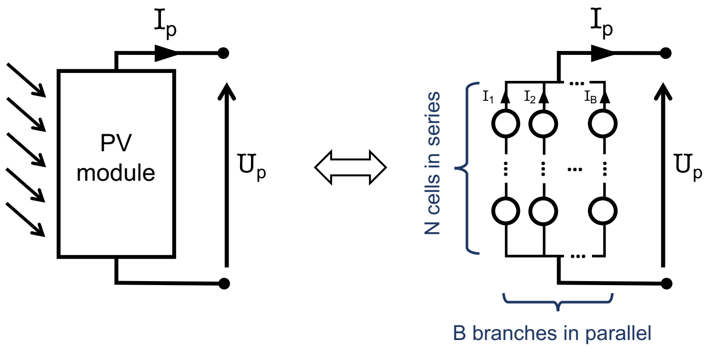
Given the pump requires the PV module to deliver 60 W to work properly, we can determine I_p given U_p=12 V:

$$U_P \cdot I_P = 60W \implies I_P = 5\ A$$

0.5pts (value of I_p)

The PV module consists of an assembly of PV cells connected in series and parallel. *To start with, let us assume that each of the parallel branches includes the same number of PV cells connected in series.*
Let N be the number of series PV cells and B the number of parallel branches.
Each branch delivers a current I_k ($k \in [1, B]$). An equivalent scheme of the PV module is depicted below:

0.5pts (assembly of identical # of cells in series + defining N and B)



In order to answer the question, one has to determine both N and B given the values of U_p and I_p.
To do so, we will assume that each individual PV cell behave similarly according to the I-V characteristic provided in Figure 2.

0.25pts (assumption of identical cells)

Therefore, the total voltage difference U_p is equally distributed along the N cells:

0.25pts (Link between U_p and U_0)

$$U_p = N \cdot U_0 \text{ (1)}$$

Where U_0 is the voltage difference across individual PV cells.

In addition, each branch would deliver the same current:

$$I_1 = I_2 = \cdots = I_B = I_0$$

Where I_0 corresponds to the current delivered by each PV cell. Given the branches are connected in parallel, the total current delivered by the PV module writes:

$$I_p = B \cdot I_0 \text{ (2)}$$

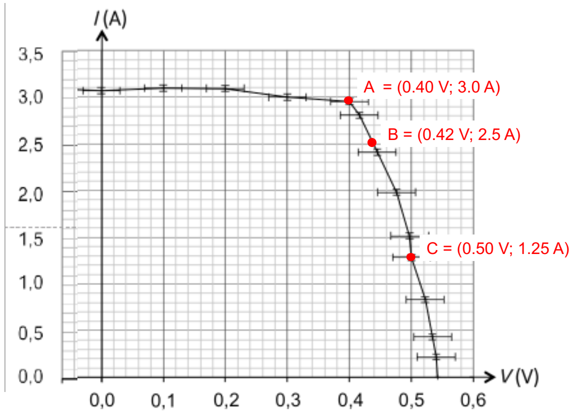
0.25pts (Link between I_p and I_0)
0.5pts (U_0 and I_0 on the IV curve)

Note that (U_0,I_0) is the *operating point of each PV cell*. Therefore <u>(U_0,I_0) should be standing on the I-V curve of the PV cell</u>. If (U_0,I_0) can be determined then, (N,B) can be found from (1) and (2).
Note that given should (U_0,I_0) lie on the I-V curve, only U_0 (or I_0) has to be determined.
(1) and (2) can be rewritten as:

$$\begin{cases} U_0 = \dfrac{U_p}{N} = \dfrac{12}{N} \\ I_0 = \dfrac{I_p}{B} = \dfrac{5}{B} \end{cases}$$

Given we have 3 unknows: B, N and U_0 (or I_0) and only 2 equations, there is no unique solution. Without further constraints, we therefore have to "try" different operating points, here by choosing deliberately (U_0,I_0) on the I-V curve (getting rid of the uncertainties), to find N and B as schematized below:

0.5pts (for choosing U_0 and I_0 on the IV curve then deducing N and B)



Choosing 3 different operating points for illustration, the number of series cells and parallel branches can be determined as follows:

(U_0, I_0)	A = (0.40V;3.0 A)	B = (0.42V; 2.5 A)	C = (0.50V;1.25 A)
$N = U_p/U_0$	30	≈28.6~29	24
$B = I_p/I_0$	≈1.7~2	2	4
Total number of cells N.B	~ 60	~ 58	96

Note that N and B have to be rounded to their closest upper values to ensure that the minimal power requirement is fulfilled.

From the above results, we can see for instance that using 96 cells organized along 4 branches of 24 PV cells operating at C = (0.5V; 1.25A) would fulfil both requirements in terms of voltage and power supply.

However, the optimal configuration should be closer from operating points A & B where only ~60 cells (N = 30, B=2) would meet the same requirements.

0.5pts (for possible correct values for N and B)

+0.5pts bonus (for other possible configurations or any statement related to optimal configurations)