INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

SCAN 1st

Mechanics test 1

Friday, 5th March 2021 – Duration: 1h30

1-page personal formula sheet authorised.

Exercise 1:

A particle is projected upwards with a speed V_0 at an angle α with the horizontal (Figure 1).

1 – Find the time and position (coordinates) when the particle will move perpendicular to its initial direction. Express the results in terms of V_0, g and α.

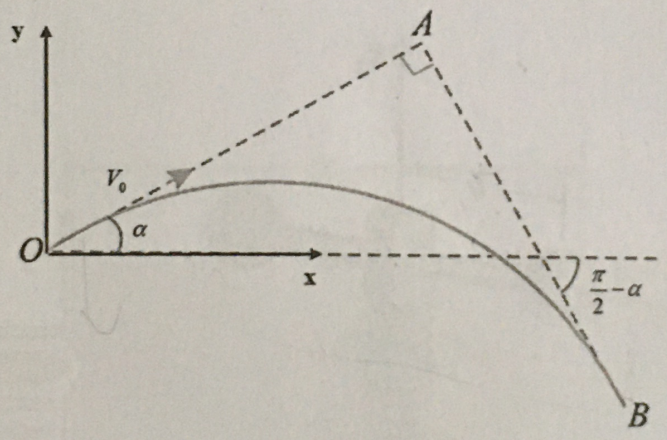

Figure 1.

Exercise 2:

The power screw in Figure 2 starts from rest and is given a rotational speed $\dot{\theta}$, which increases uniformly with time so that:

$\frac{d\theta}{dt} = \dot{\theta} = kt$, where k is a constant

The (constant) lead of the screw (advancement per revolution) is L so that the axial position of the screw is defined by:

$z = \frac{L}{2\pi}\theta + z_0$, where z_0 is a constant (initial axial position)

1 – Using cylindrical coordinates, determine the velocity and acceleration vectors of the centre of ball A located at distance b from the screw axis (Figure 2).

2 – Calculate the speed (magnitude of the velocity vector) and deduce the unit tangent vector to the trajectory. Verify that its orientation is constant with respect to the cylindrical coordinate system. Deduce the nature of the trajectory of point A with respect to the ground (no equation required)?

3 – From the expressions of the acceleration vector and the tangent vector, deduce the normal vector to the trajectory and the radius of curvature of the trajectory.

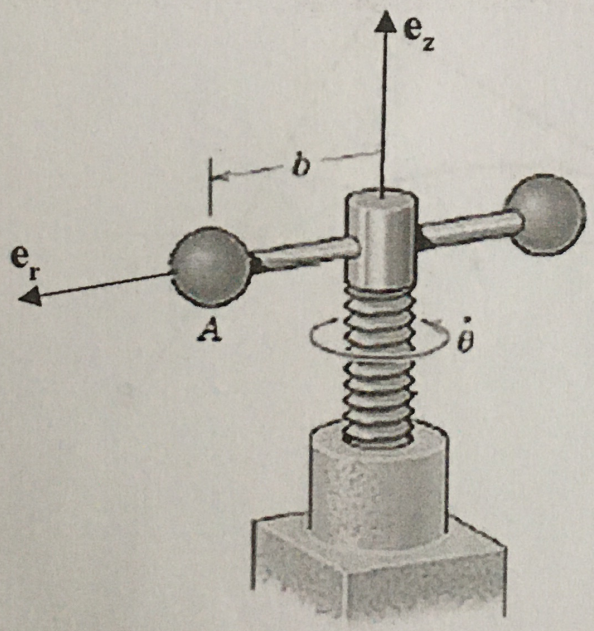

Figure 2.

Exercise 3:

A test projectile is fired horizontally into a viscous liquid with an initial speed v_0 (Figure 3). The retarding force is proportional to the square of the speed, so that the horizontal acceleration of the particle is $a = -k\,v^2$.

1 – By integration of the formula $a\,ds = v\,dv$ for rectilinear motions, determine the distance D travelled in the liquid to reduce the speed to $\frac{v_0}{2}$. Specify all the hypotheses used.

2 – Deduce the corresponding time T of travel (to reduce the speed to half its initial value) by integration of $a = \frac{dv}{dt}$.

3 – Is it possible to determine the maximum distance travelled by the projectile by extrapolating the results in question 1? How would you formulate the problem if you had to find this maximum distance?

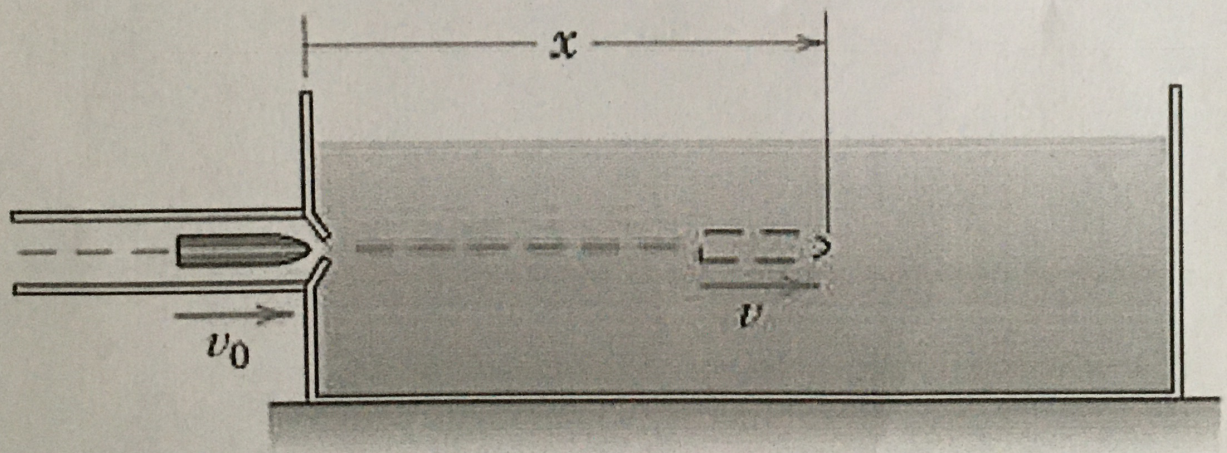

Figure 3.