Physics test - Electromagnetism
June 14th 2022
Duration : 2h00
non programmable calculator allowed
no documents allowed

The marks will account not only for the results, but also for the justifications, and the way you analyze the results. Moreover, any result must be given in its literal form involving only the data given in the text. It is also reminded that the general clarity and cleanness of your paper may also be taken into account.

Exercise 1: Electromagnetic Train ($\sim$ 4 pts)

A simple electromagnetic "train", shown in Fig. 1, can be fabricated from the following components :

- a conducting wire wound into the shape of a solenoid (coil)
- disk-shaped magnets for which:
    - the North and South poles are on opposite flat faces of the disk
    - the material used is an electrical conductor
- an AA battery (to which the magnets are attached as shown in Fig. 1).

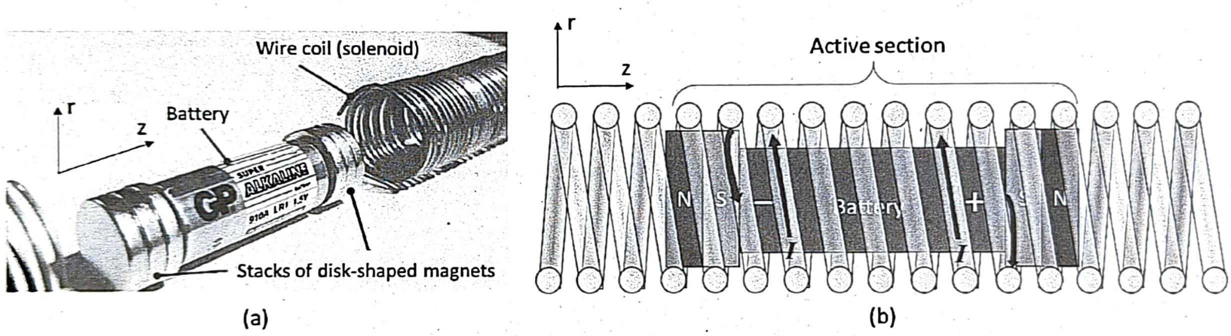


Figure 1: (a) Composition of electromagnetic "train", (b) schematic representation of battery and magnet assembly (*i.e.* the train) and solenoid, with the current flow from positive to negative terminal of battery through the solenoid indicated by black arrows.

When the battery and magnet assembly ("train") is inserted into the solenoid, such that the electrical poles of the battery make contact with the conducting wire, a current flows through the "active section" of solenoid as shown in figure 1(b). As a result of this current, the train is propelled along the long axis of the solenoid until it is ejected at the far end, as shown in the time-lapse photos of Fig. 2.

In the following questions, we will study qualitatively the phenomena behind the motion observed.

1. Using the cross and dot representation, complete the diagram in Appendix 1 (page 7) to show the direction of current flow (into or out of the page) for the wires cut by the plane of the diagram.

2. Draw in Appendix 1 (page 7) the orientated field lines associated with the magnetic fields produced by the magnet $M1$, the solenoid and the magnet $M2$ in regions 1,2 and 3 respectively. We note that the magnetic fields from the solenoid and the magnets overlap in reality, despite us having drawn them in distinct regions of space for clarity.

3. Determine the magnetic poles of the individual loop of the solenoid shown in the dashed box in region 2. Write your answer "N" or "S" in boxes $P1$ and $P2$.

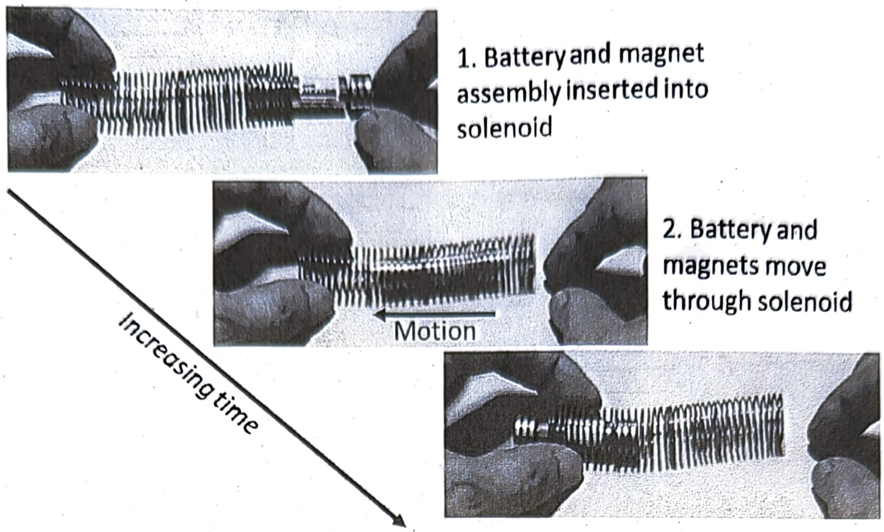


Figure 2: Time-lapse photos of the "train" showing motion through the solenoid. The solenoid is held in a fixed position.

4. Reasoning only in terms of attractive or repulsive forces between magnetic poles, describe how the train advances through the solenoid. For the configuration shown in Appendix 1 (page 7), will the train move to the left of the right ?

5. What would happen if only magnet $M1$ was reversed (north and south poles switched) ?

Exercise 2: The Current Clamp ($\sim$ 10 pts)

A current clamp (Fig. 3-a) can be used to measure electrical currents without having to break a circuit, ensuring for instance safety and continuity of service. Another of its advantage is that it can be used to measure large current intensities (> 100 A).

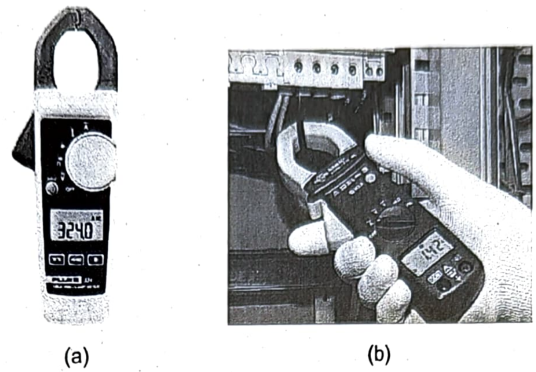


Figure 3: (a) Current clamp. (b) Clamp under measurement

For this, the clamp has to surround the wire on which the measurement is being made; the current intensity running through the wire can be then directly read on a screen (Fig. 3-b). In this exercise, we will study the working principle of the current clamp, which is based on induction phenomena, through a simplified model.

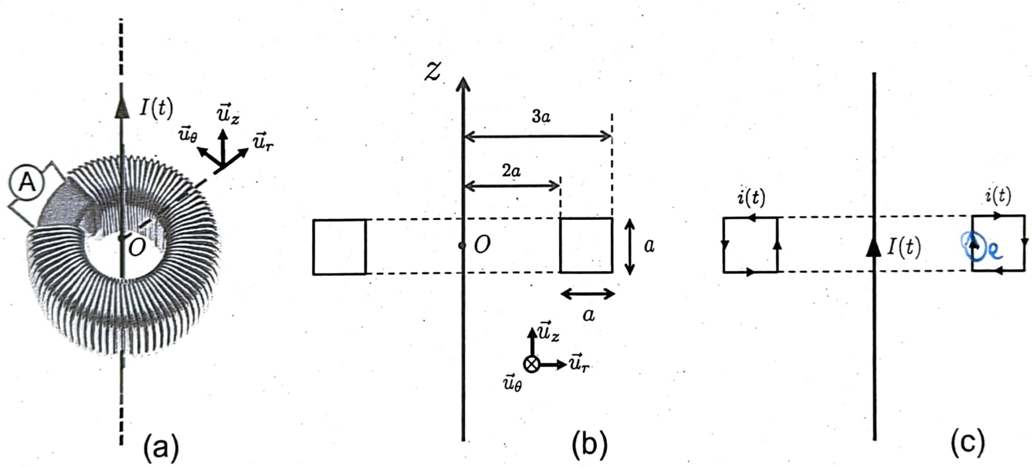


Figure 4: (a) Schematic view of the measuring coil closed by an ammeter. Cross section view of the clamp showing its geometry (b) and the different current intensities (c).

The current clamp can be described as torus coil having a square section, side a, and revolution axis Oz (Fig. 4-a&b), on which a wire is wound, thus forming N square turns arranged in series. The turns are situated at an average distance of $\frac{5}{2}a$ from (Oz) (Fig. 4-b). The coil surrounds an infinite wire in which a current $I(t)$ is running along (Oz). The total resistance of the coil is denoted R, it is connected in series to an internal ammeter (negligible resistance, Fig. 4-a) closing the circuit and measuring the current $i(t)$ running through it (Fig. 4-c).
In the following we will consider that an AC current $I(t)$ is running through the wire with:

$$I(t) = I_0 \cos(\omega t)$$

<u>Numerical values</u>: $N = 1000$, $a = 1$ cm, $R = 5\ \Omega$, $\omega = 100\pi$

Throughout this exercise we will consider the cylindrical frame $(\vec{u}_r, \vec{u}_\theta, \vec{u}_z)$ as denoted in Figure 4.

In this exercise we will show that the total current $i(t)$ flowing into the current clamp originates from both the effect of the $\vec{B}$ field of the wire on the coil and from the self-induction phenomenon.

Effect of the wire on the coil

1. We denote $\vec{B}_w$ the magnetic field created by the wire at any point of space $P = (r, \theta, z)$:

$$\vec{B}_w(P) = \frac{\mu_0 I(t)}{2\pi r}\vec{u}_\theta$$

where $\mu_0 = 4\pi\, 10^{-7}$ H/m is the vacuum permeability.

a) Briefly justify the expression of $\vec{B}_w$ using simple geometrical arguments (*i.e.* symmetries/antisymmetries and invariances).

b) Do a sketch of the field lines of $\vec{B}_w$ in a plane perpendicular to (Oz) containing O making apparent the boundaries of the coil.

2. We consider a single turn of the coil and P a point situated on the surface delimited by its boundaries (thus $2a \leq r \leq 3a$ and $-a/2 \leq z \leq a/2$). The surface of the coil will be oriented using the current intensity $i(t)$.

a) According to which vector is directed the normal to the coil's surface?

b) Determine the elementary magnetic flux $d\Phi_w^1(P)$ of $\vec{B}_w$ at P.

c) Calculate the magnetic flux through a single turn Φ_w^1.

3. Show that the total flux of $\vec{B}_w$ through the coil writes $\Phi_w = M \cdot I(t)$ with $M = \frac{\mu_0 N}{2\pi} a \ln\left(\frac{5}{2}\right)$. What is the physical meaning of M? Give its numerical value.

4. Briefly explain why an electrical current can be expected to run inside the coil.

Accounting for self-induction

The current intensity $i(t)$ running inside the coil induces a magnetic field $\vec{B}_c$. For the sake of simplicity, we will assume that the magnitude of $\vec{B}_c$ is homogeneous and constant inside the coil and is worth $||\vec{B}_c|| = \frac{\mu_0 N i(t)}{5\pi a}$.

5. For what reason(s) do we have have to account for self induction?

6. Determine the orientation of $\vec{B}_c$ inside the coil using any physical arguments you would find relevant.

7. Determine the self-magnetic flux Φ^s inside the coil (N turns) and show that it can be written as $\Phi^s = L \cdot i(t)$. Determine L as function of μ_0, N and a. What is the physical meaning of L? Give its numerical value.

Finding the current intensity I_0

8. Determine the total magnetic flux ϕ_{tot} through the coil and deduce the expression of the induced electromotive force $e(t)$ inside the coil in terms of L, M and the time derivatives of I and i.

9. Give an equivalent electrical scheme of the current clamp featuring the e.m.f. $e(t)$ and its orientation (Given its negligible resistance the ammeter can be considered as a short-circuit). Deduce a differential equation based on $i(t)$ in which the second member holds $\frac{dI(t)}{dt}$.

Let i_0 be the amplitude of $i(t)$ in AC steady-state. We remind then that using complex notations $i(t)$ and $I(t)$ write:

$$\underline{I}(t) = I_0\, e^{j\omega t} \text{ and } \underline{i}(t) = i_0\, e^{j(\omega t + \varphi)}$$

where φ denotes the phase-shift between $i(t)$ and $I(t)$ that we won't study in the following.

The series ammeter in the clamp gives $i_0 = 11$ mA

10. From the previous question, find the expression of I_0 as function i_0, M, L, R and ω and determine the value of I_0.

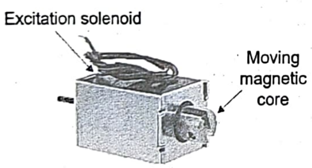
BONUS: Is it be possible to measure DC currents with the current clamp?

Exercise 3: Electromagnetic actuator ($\sim$ 6 pts)

In this study, an **electromagnetic actuator,** *i.e.*, a component producing a motion (linear in this case) of a moveable element upon application of an electrical stimulus, is investigated. The actuator, shown in Fig. 5, is composed of an **excitation solenoid**, also referred as **excitation coil**, within a frame and a **movable magnetic core** lying **inside** the excitation coil. A **compression spring** (which is **non-magnetic**) is attached to one end of the magnetic core to exert a restoring force on it (Fig. 5(b)). When a **current i_s is applied to the excitation solenoid,** it magnetizes the moving magnetic core which then behaves like a magnet: the **resulting interaction** force between the two magnetic elements (*i.e.*, the excitation coil and the magnetic core) then induces a displacement of the magnetic core. In the following this force will be referred to as the **magnetic force**. The objective of the present study consists in investigating the mechanical actuation performance. The final aim is to check the values given in the manufacturer datasheet (Fig. 5(a)).

To establish a model of the actuator, the **magnetized moving magnetic core will also be considered as a solenoid**, with a flowing **current i_c** linked to i_s (as the latter is the origin of the magnetic field in the excitation solenoid). The resulting model is given in Fig. 6.

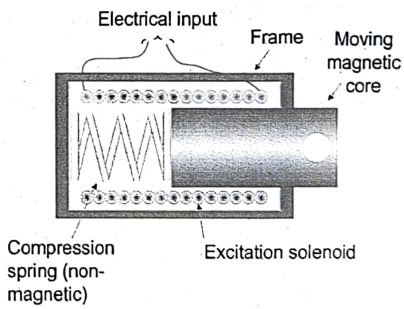
Additionally, the following points are considered:
- The **stiffness** of the spring is denoted K.
- The excitation solenoid and the moving core magnet have similar circular **cross-sections** S.
- The excitation solenoid is sufficiently long to ensure that the **left side of the moving magnetic core never** exits from any side of the excitation coil. Also, the **magnetic core never fully enters** the excitation coil.
- **At rest**, the excitation solenoid and moving magnetic core **overlap by a distance l.**
- The relationship between the excitation coil current i_s and the equivalent current i_c of the solenoid modeling the magnetized moving core is given as: $i_c = \gamma i_s$ ($\gamma > 0$).
- The **number of turns per unit length of the equivalent solenoid** representing the moving magnetic core is denoted as n_c.
- The axis of the solenoids is denoted x, and the origin O is at the right end of the excitation solenoid.
- The **displacement (positive to the left)** of the moving magnetic core is denoted as u (so that the **force**



Specifications

Attribute	Value
Supply Voltage	24 V dc
Action	Push Pull
Maximum Stroke	10mm
Holding Force	5N
Dimensions	45 x 31.8 x 23.4

(a)



(b)

Figure 5: Actuator (a) overview and characteristics (source: `https://fr.rs-online.com`) and (b) inner view.

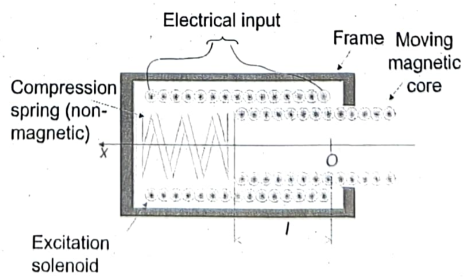


Figure 6: Actuator model.

Parameter	Value
Stiffness K	500 N.m^{-1}
Excitation solenoid/moving magnetic core cross-section S	5 cm^2
Excitation coil number of turns per unit length n_s	3 000 turns.m^{-1}
Equivalent solenoid for the moving magnetic core number of turns per unit length n_c	10 000 turns.m^{-1}
Magnetization parameter γ	10
Excitation coil maximal current i_s	5 A

Table 1: Additional actuator characteristics and numerical values of parameters.

exerted by the spring has a **magnitude of** $K.u$).

- The **magnetic force** is directed **along** (Ox) and F_{mag} denotes its **algebraic** component along (Ox).
- **Weight is neglected** and **no friction** is assumed.
- Additional parameter values are reported in **Table 1** for numerical applications.

When a **current flows through the excitation solenoid,** it induces a **magnetic field** B_s, which is considered to be uniform and within the excitation solenoid, and vanishingly small (or "nil") outside. It is then defined as:

$$\vec{B}_s = \begin{cases} \mu_0 n_s i_s \, \vec{u}_x & \text{for } x \geq 0 \\ \vec{0} & \text{for } x < 0 \end{cases} \tag{1}$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m is the vacuum permeability and n_s the number of turns per unit length for the excitation solenoid. Note that as the moving magnetic core never exits from the left side of the excitation coil, there is no need to define B_s for sufficiently large x values.

1. Make a **sketch** of the device by **defining all the necessary variables and parameters**. Define the **conventional current direction for both the excitation solenoid and the magnetic core,** so that the normal vectors to the associated surfaces are oriented **towards** (Ox). Define the **system** and represent all **forces** acting on it (it is recalled that weight can be neglected, that no friction is considered and that the magnetic force is assumed to be oriented along (Ox)).

2. In this question, we investigate the magnetic **flux** through the magnetic core.
   a) What is the **overlap** distance between the excitation coil and the moving magnetic core when the latter is **moved to the left by a distance** u?
   b) **How many turns** N_c of the equivalent solenoid representing the **moving magnetic core** are **within the excitation coil** (as a function of the problem parameters)? Give N_c in terms of n_c, l and u.
   c) Deduce the **total magnetic flux** Φ_c induced by the excitation coil through the moving magnetic core (the self-flux originating from i_c will be neglected).
   d) Determine the associated magnetic potential energy variation $\mathrm{d}\, E_p|_{mag}$ of the magnetic core (still neglecting its self-flux) associated with a variation "du" of the displacement.

3. Next, we investigate the **magnetic force** exerted on the movable magnetic core.
   a) From the previous expression and noting that no energy loss is considered in the system, deduce the **magnetic force** F_{mag} exerted on the moving magnetic core as a function of the **current** i_s, **permeability** μ_0 **and other parameters of Table1**. Conclusion? *Hint: we can consider the work of this force for a small change "du" in the displacement.*
   b) Calculate the **value of** F_{mag} using the parameters given in Table 1. Compare with the **characteristics of the datasheet** given in Fig. 5(a).

4. Finally, we aim at retrieving the **maximal displacement.**
   a) From the analysis of the **force balance at equilibrium**, express the **displacement** u as a function of the **excitation current** i_s.
   b) From the device characteristics given in Table 1, what is the **maximum displacement** of the actuator? **Compare with the datasheet** given in Fig. 5(a).